Lecture 12

2023/2024

Microwave Devices and Circuits for Radiocommunications

2023/2024

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 16-18, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 20-27.02.2024 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

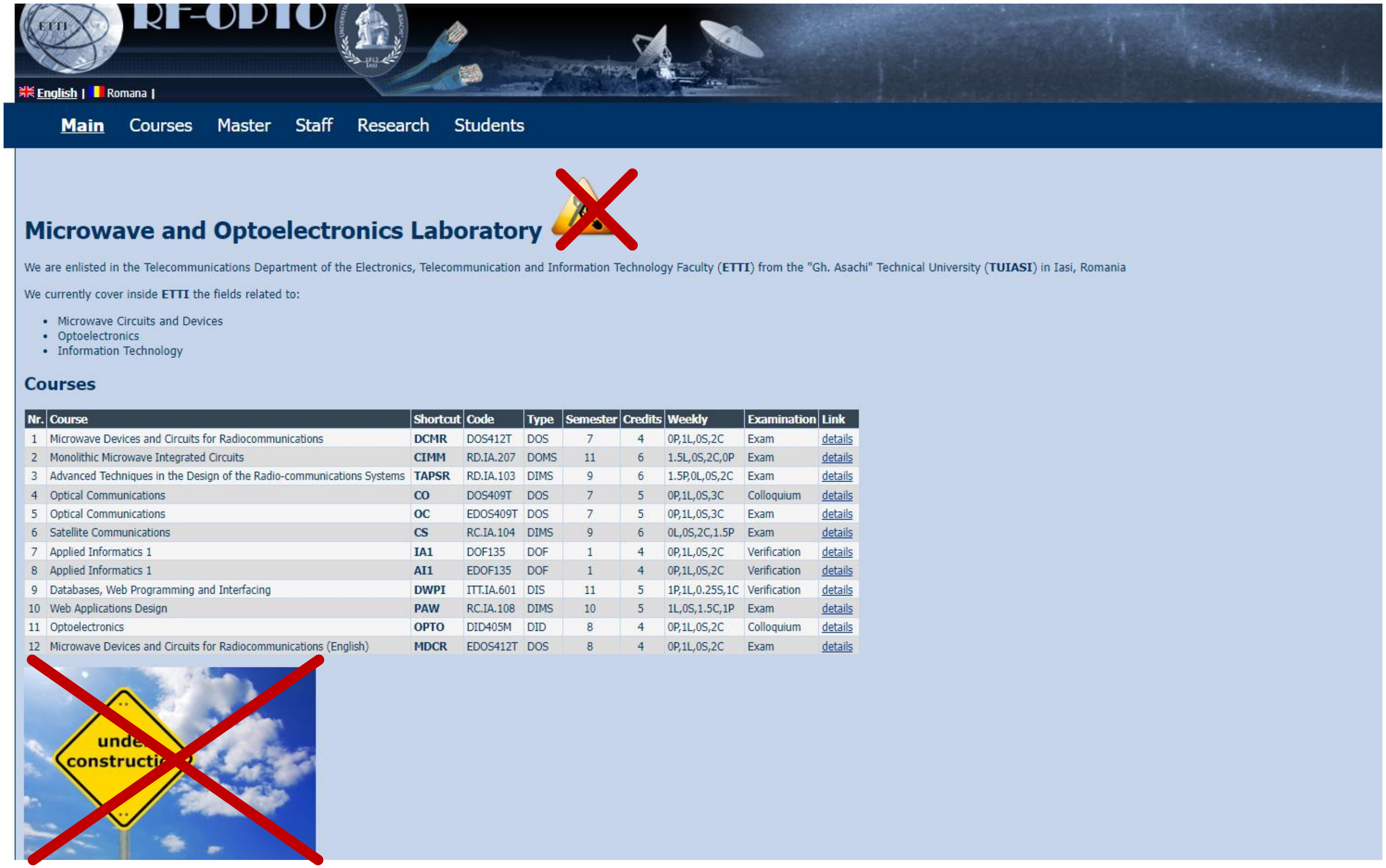
2023/2024

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 20.02.2024)
    - personal homework

Materials

- http://rf-opto.etti.tuiasi.ro

Site



English | Romana |

Main Courses Master Staff Research Students

Microwave and Optoelectronics Laboratory

We are enlisted in the Telecommunications Department of the Electronics, Telecommunication and Information Technology Faculty (**ETTI**) from the "Gh. Asachi" Technical University (**TUIASI**) in Iasi, Romania

We currently cover inside **ETTI** the fields related to:

- Microwave Circuits and Devices
- Optoelectronics
- Information Technology

Courses

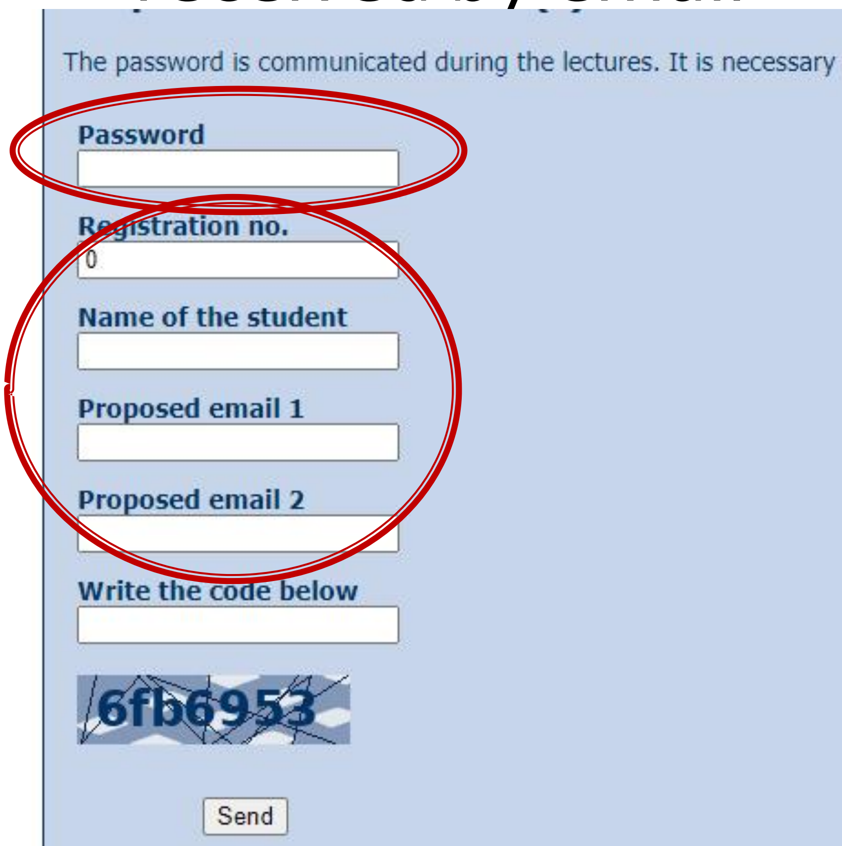
Nr.	Course	Shortcut	Code	Type	Semester	Credits	Weekly	Examination	Link
1	Microwave Devices and Circuits for Radiocommunications	**DCMR**	DOS412T	DOS	7	4	0P,1L,0S,2C	Exam	details
2	Monolithic Microwave Integrated Circuits	**CIMM**	RD.IA.207	DOMS	11	6	1.5L,0S,2C,0P	Exam	details
3	Advanced Techniques in the Design of the Radio-communications Systems	**TAPSR**	RD.IA.103	DIMS	9	6	1.5P,0L,0S,2C	Exam	details
4	Optical Communications	**CO**	DOS409T	DOS	7	5	0P,1L,0S,3C	Colloquium	details
5	Optical Communications	**OC**	EDOS409T	DOS	7	5	0P,1L,0S,3C	Exam	details
6	Satellite Communications	**CS**	RC.IA.104	DIMS	9	6	0L,0S,2C,1.5P	Exam	details
7	Applied Informatics 1	**IA1**	DOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
8	Applied Informatics 1	**AI1**	EDOF135	DOF	1	4	0P,1L,0S,2C	Verification	details
9	Databases, Web Programming and Interfacing	**DWPI**	ITT.IA.601	DIS	11	5	1P,1L,0.25S,1C	Verification	details
10	Web Applications Design	**PAW**	RC.IA.108	DIMS	10	5	1L,0S,1.5C,1P	Exam	details
11	Optoelectronics	**OPTO**	DID405M	DID	8	4	0P,1L,0S,2C	Colloquium	details
12	Microwave Devices and Circuits for Radiocommunications (English)	**MDCR**	EDOS412T	DOS	8	4	0P,1L,0S,2C	Exam	details

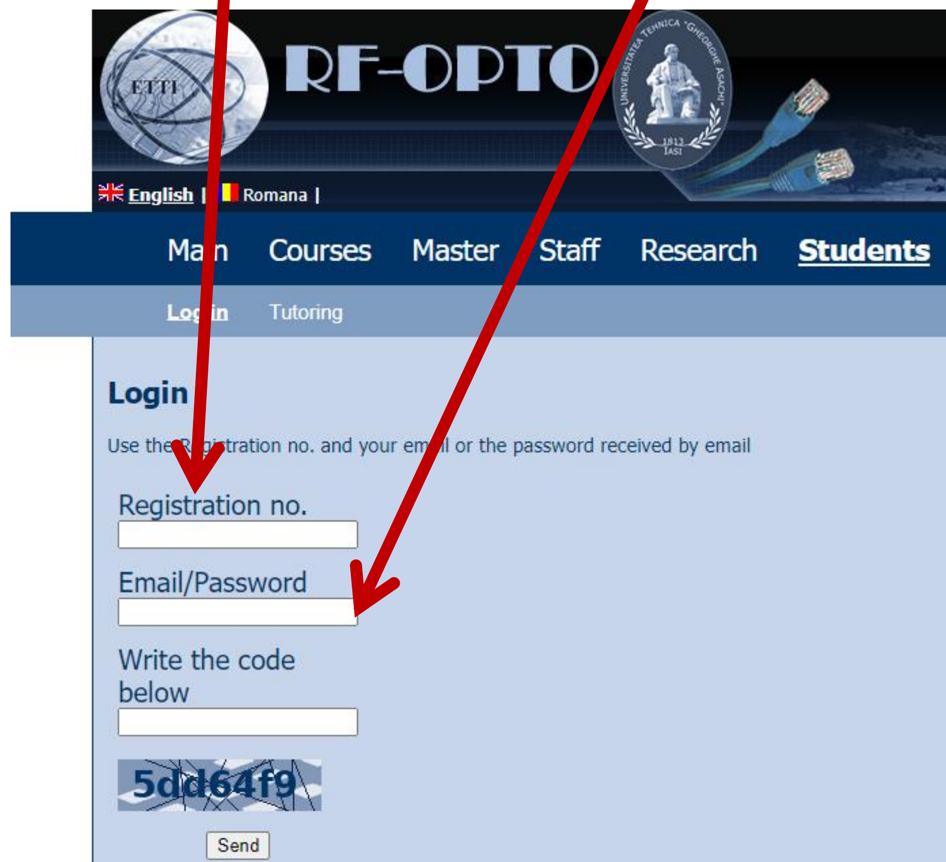
Materials

- RF-OPTO
  - http://rf-opto.etti.tuiasi.ro
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/**online exam > Week4-Week6**
  - used at lectures/laboratory

Online – Registration no.

- access to **online exams** requires the **password** received by email





Password

- received by email

Important message from RF-OPTO Inbox ×

Radu-Florin Damian
to me, POPESCU

Romanian > English Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

---

Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply Reply all Forward

Subject	Correspondents
Important message from RF-OPTO	POPESCU GOPO ION
Validation of MDCR exam from 02/05/2020	[redacted]
[redacted]	[redacted]

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

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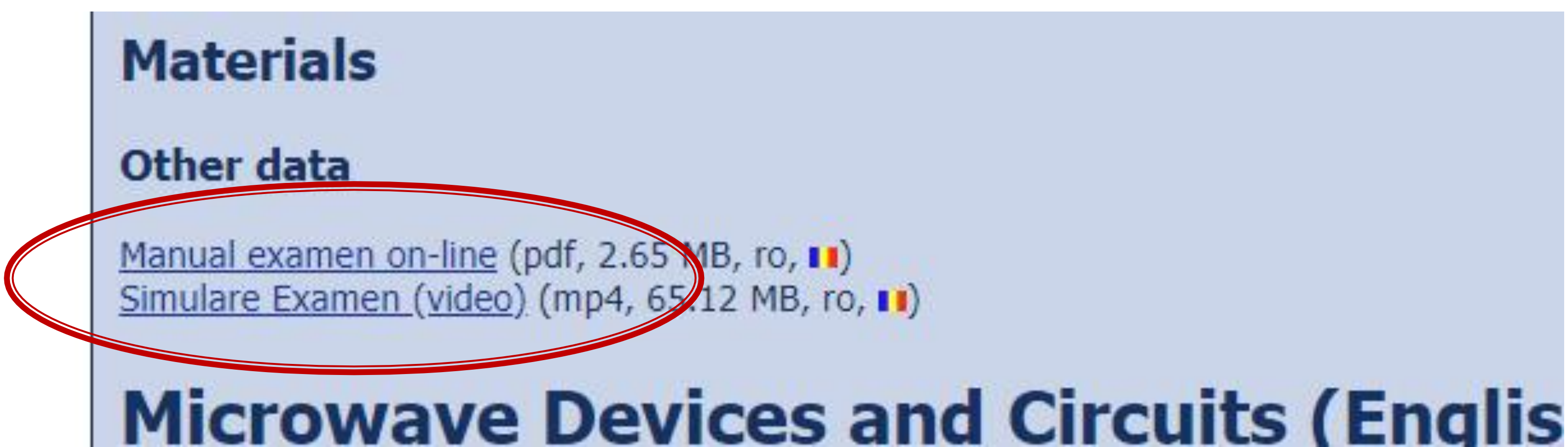
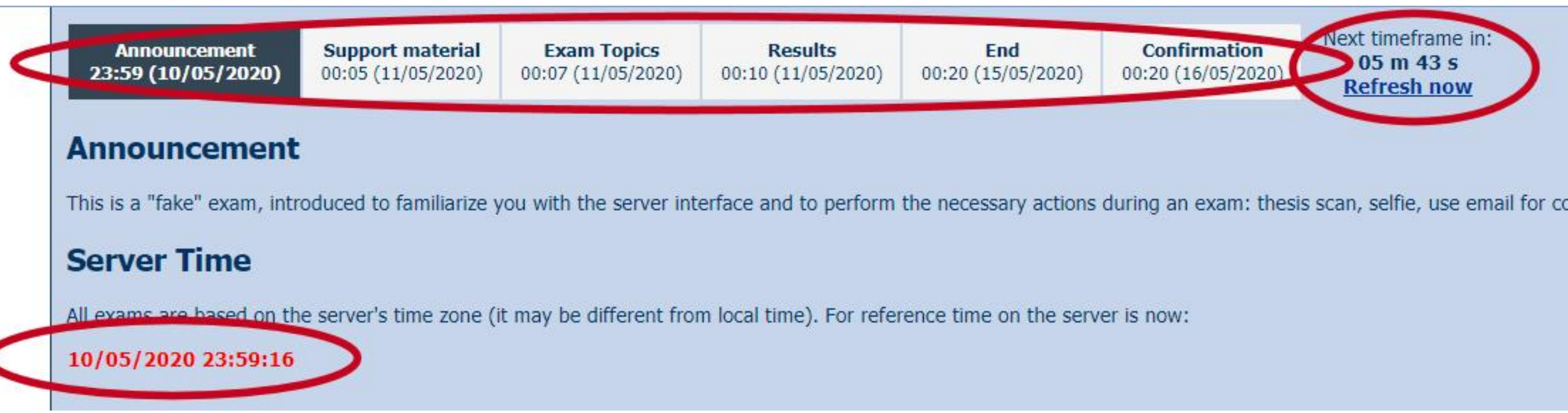
Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~



Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri s	T2, fisier parmetri s	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

Online results submission

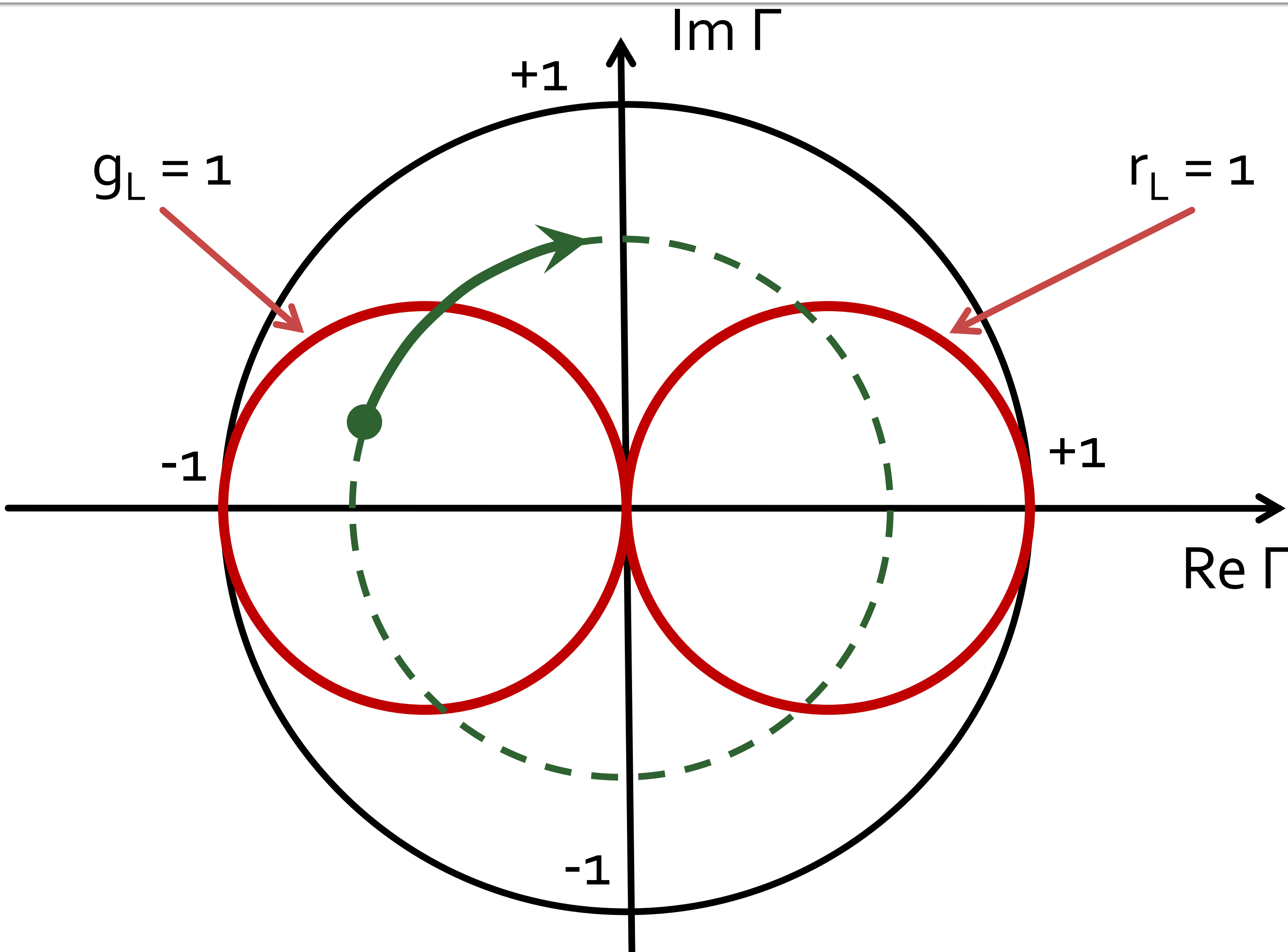
Grade = Quality of the work +

+ Quality of the submission

Impedance Matching

Impedance Matching with Stubs

Smith chart, r=1 and g=1

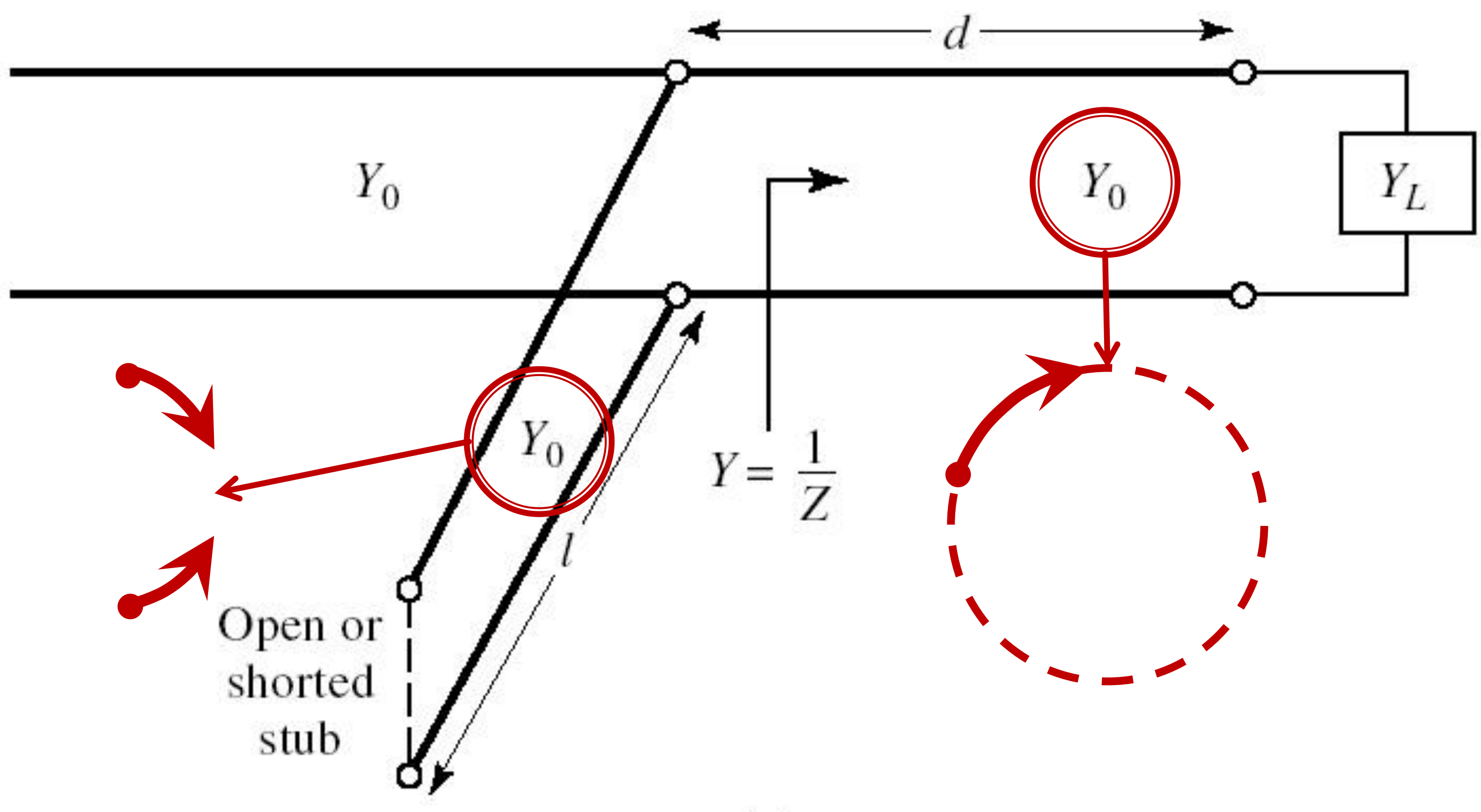


Analytical solutions

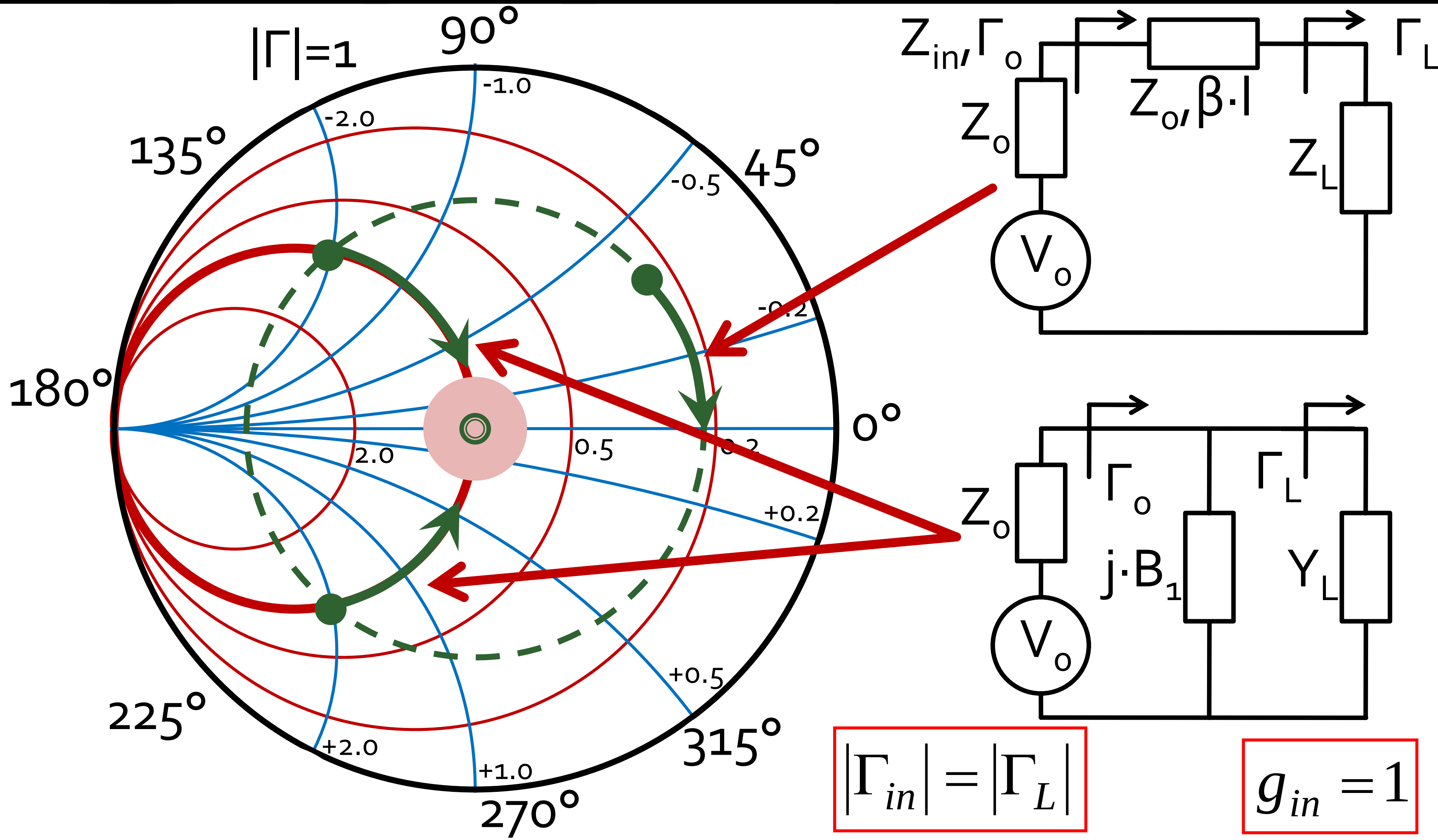
Exam / Project

Case 1, Shunt Stub

- Shunt Stub



Matching, series line + shunt susceptance



$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$g_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = -|\Gamma_S|$$

$$\theta_{sp} = \beta \cdot l = \tan^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.593\angle 46.85°$$

$$|\Gamma_S| = 0.593; \quad \varphi = 46.85° \qquad \cos(\varphi + 2\theta) = -0.593 \Rightarrow (\varphi + 2\theta) = \pm 126.35°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation
  - **"+" solution**

    $$(46.85° + 2\theta) = +126.35° \qquad \theta = +39.7° \qquad \operatorname{Im} y_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.472$$

    $$\theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = -55.8°(+180°) \rightarrow \theta_{sp} = 124.2°$$

  - **"-" solution**

    $$(46.85° + 2\theta) = -126.35° \qquad \theta = -86.6°(+180°) \rightarrow \theta = 93.4°$$

    $$\operatorname{Im} y_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.472 \qquad \theta_{sp} = \tan^{-1}(\operatorname{Im} y_S) = 55.8°$$

Analytical solution, usage

$$(\varphi+2\theta)=\begin{cases}+126.35^\circ\\-126.35^\circ\end{cases}\quad \theta=\begin{cases}39.7^\circ\\93.4^\circ\end{cases}\quad \mathrm{Im}[y_S(\theta)]=\begin{cases}-1.472\\+1.472\end{cases}\quad \theta_{sp}=\begin{cases}-55.8^\circ+180^\circ=124.2^\circ\\+55.8^\circ\end{cases}$$

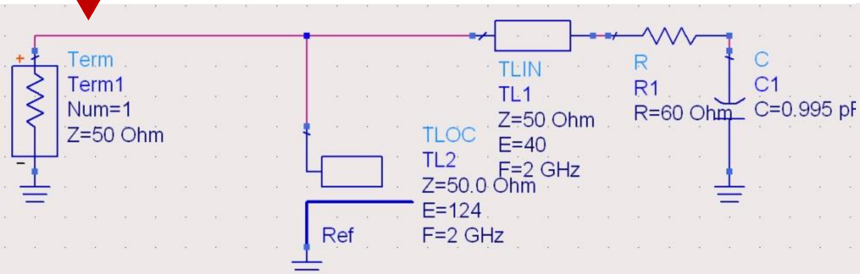
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **shunt stub** equation

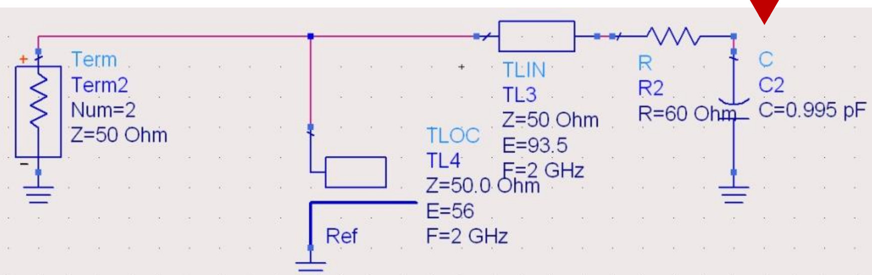
$$l_1=\frac{39.7^\circ}{360^\circ}\cdot\lambda=0.110\cdot\lambda$$

$$l_2=\frac{124.2^\circ}{360^\circ}\cdot\lambda=0.345\cdot\lambda$$

$$l_1=\frac{93.4^\circ}{360^\circ}\cdot\lambda=0.259\cdot\lambda$$

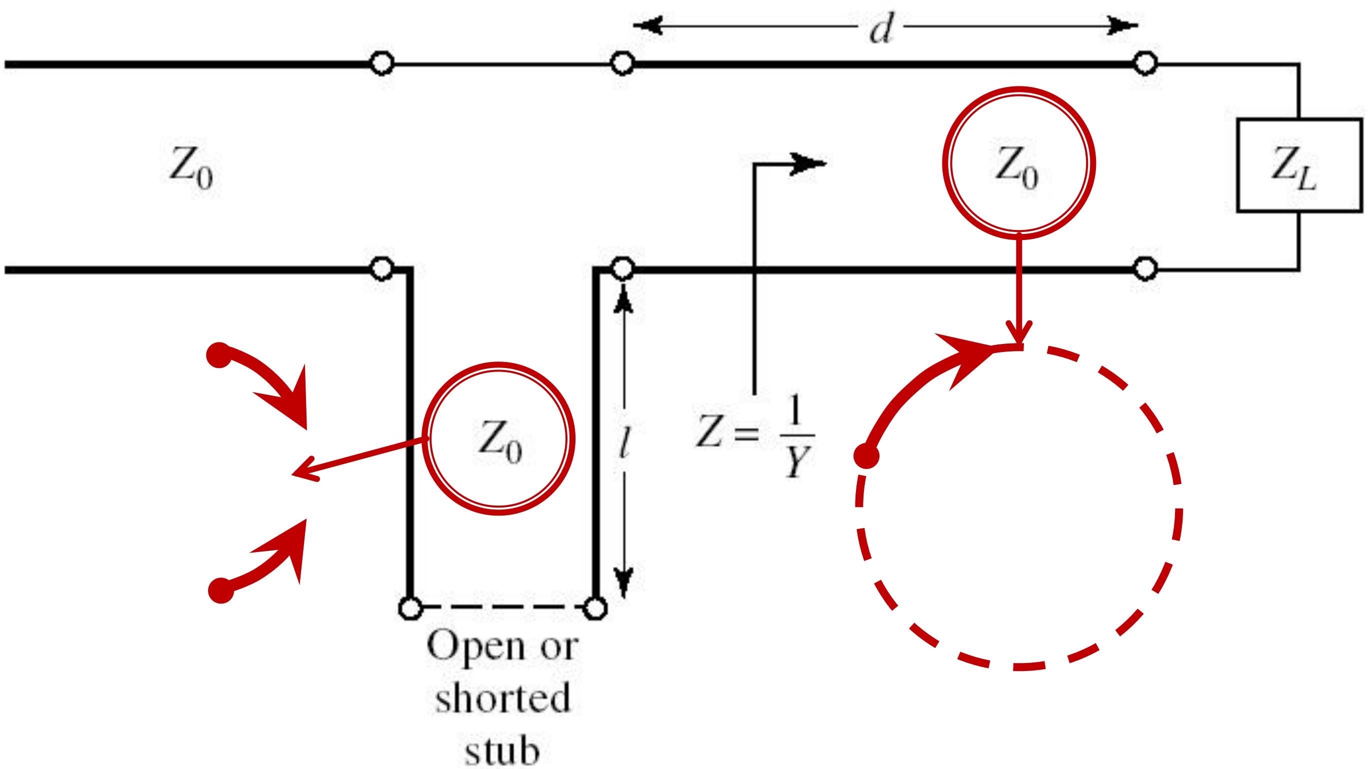
$$l_2=\frac{55.8^\circ}{360^\circ}\cdot\lambda=0.155\cdot\lambda$$



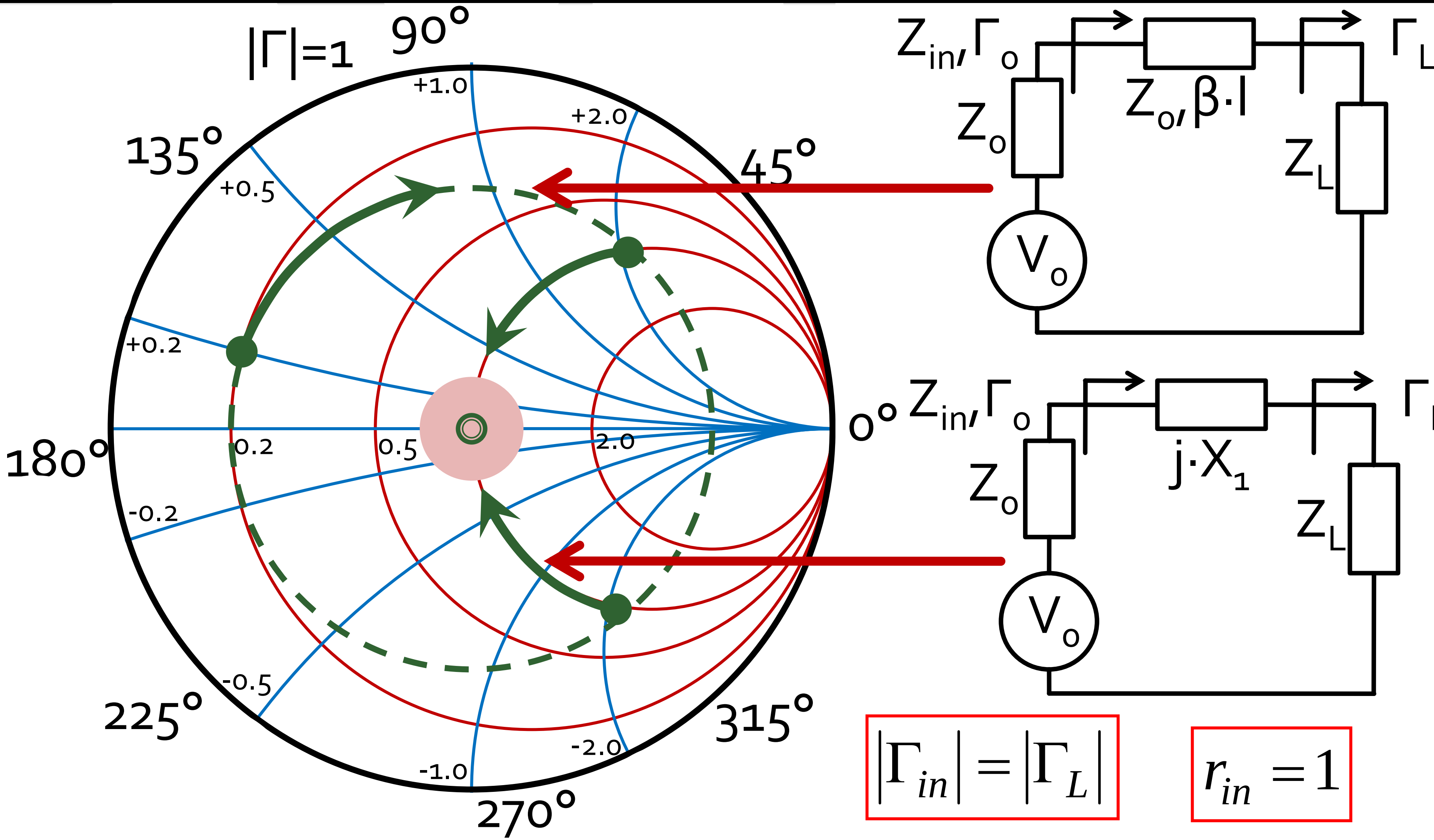


Case 2, Series Stub

- Series Stub
- difficult to realize in single conductor line technologies (microstrip)



Matching, series line + series reactance



$$\left|\Gamma_{in}\right| = \left|\Gamma_L\right|$$

$$r_{in} = 1$$

Analytical solution, usage

$$\cos(\varphi + 2\theta) = |\Gamma_S|$$

$$\theta_{ss} = \beta \cdot l = \cot^{-1} \frac{\mp 2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}}$$

$$\Gamma_S = 0.555 \angle -29.92°$$

$$|\Gamma_S| = 0.555; \quad \varphi = -29.92° \qquad \cos(\varphi + 2\theta) = 0.555 \Rightarrow (\varphi + 2\theta) = \pm 56.28°$$

- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation
  - **"+" solution**

$$(-29.92° + 2\theta) = +56.28° \qquad \theta = 43.1° \qquad \operatorname{Im} z_S = \frac{+2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = +1.335$$

$$\theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = -36.8°(+180°) \rightarrow \theta_{ss} = 143.2°$$

  - **"-" solution**

$$(-29.92° + 2\theta) = -56.28° \qquad \theta = -13.2°(+180°) \rightarrow \theta = 166.8°$$

$$\operatorname{Im} z_S = \frac{-2 \cdot |\Gamma_S|}{\sqrt{1 - |\Gamma_S|^2}} = -1.335 \qquad \theta_{ss} = -\cot^{-1}(\operatorname{Im} z_S) = 36.8°$$

Analytical solution, usage

$$(\varphi+2\theta)=\begin{cases}+56.28^\circ\\-56.28^\circ\end{cases}\quad \theta=\begin{cases}43.1^\circ\\166.8^\circ\end{cases}\quad \mathrm{Im}[z_S(\theta)]=\begin{cases}+1.335\\-1.335\end{cases}\quad \theta_{ss}=\begin{cases}-36.8^\circ+180^\circ=143.2^\circ\\+36.8^\circ\end{cases}$$

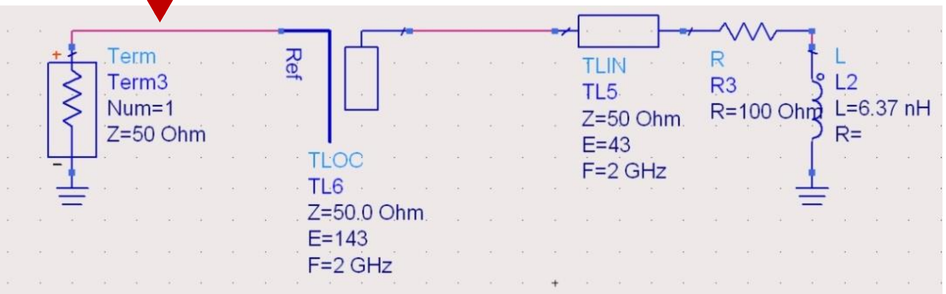
- We choose **one** of the two possible solutions
- The **sign** (+/-) chosen for the **series line** equation imposes the **sign** used for the **series stub** equation

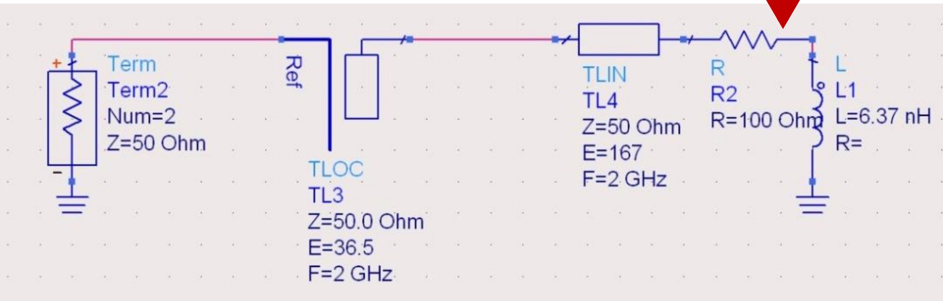
$$l_1=\frac{43.1^\circ}{360^\circ}\cdot\lambda=0.120\cdot\lambda$$

$$l_2=\frac{143.2^\circ}{360^\circ}\cdot\lambda=0.398\cdot\lambda$$

$$l_1=\frac{166.8^\circ}{360^\circ}\cdot\lambda=0.463\cdot\lambda$$

$$l_2=\frac{36.8^\circ}{360^\circ}\cdot\lambda=0.102\cdot\lambda$$





Stub, observations

- adding or subtracting **180°** (λ/2) doesn't change the result (full rotation around the Smith Chart)

$$E = \beta \cdot l = \pi = 180^\circ \qquad l = k \cdot \frac{\lambda}{2}, \forall k \in \mathbf{N}$$

  - if the lines/stubs result with **negative** "length"/ "electrical length" we add λ/2 / 180° to obtain physically realizable lines
- adding or subtracting **90°** (λ/4) change the stub impedance:

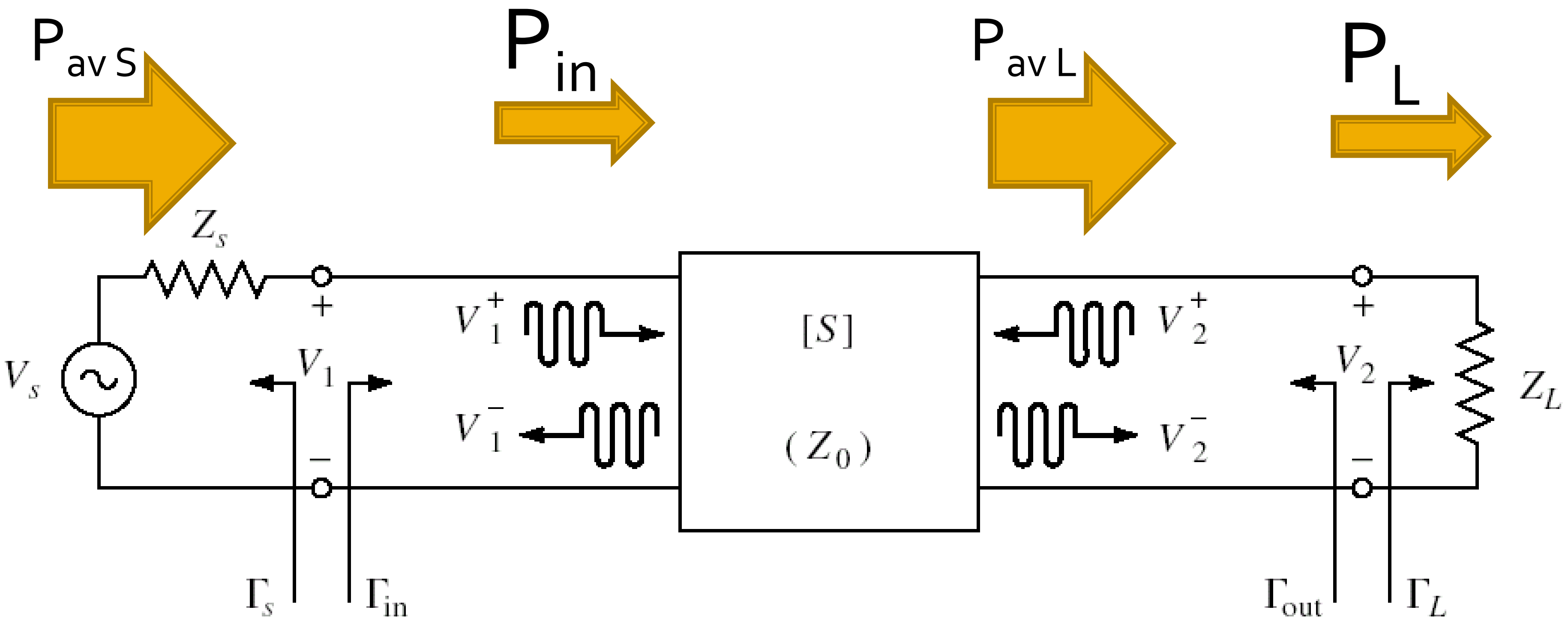
$$Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \quad \Leftrightarrow \quad Z_{in,g} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

  - for the stub we can add or subtract 90° (λ/4) while in the same time changing **open-circuit** ⇔ **short-circuit**

Microwave Amplifiers

Power / Matching

- Two ports in which matching influences the power transfer



Two-Port Power Gains

- **Available** power gain

$$G_A = \frac{P_{av\,L}}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)}{|1-S_{22}\cdot\Gamma_L|^2 \cdot \left(1-|\Gamma_{out}|^2\right)}$$

- **Transducer** power gain

$$G_T = \frac{P_L}{P_{av\,S}} = \frac{|S_{21}|^2 \cdot \left(1-|\Gamma_S|^2\right)\cdot\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\cdot\Gamma_{in}|^2 \cdot |1-S_{22}\cdot\Gamma_L|^2}$$

$$\Gamma_{in} = \Gamma_{in}(\Gamma_L)$$

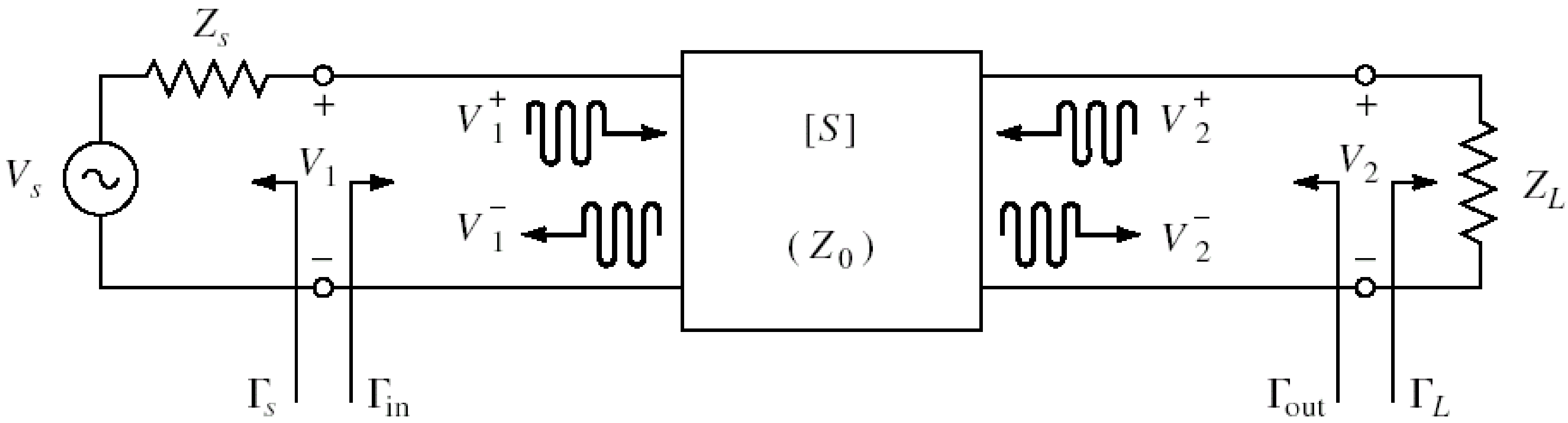
- **Unilateral transducer** power gain

$$G_{TU} = |S_{21}|^2 \cdot \frac{1-|\Gamma_S|^2}{|1-S_{11}\cdot\Gamma_S|^2} \cdot \frac{1-|\Gamma_L|^2}{|1-S_{22}\cdot\Gamma_L|^2}$$

$$S_{12} \cong 0 \qquad \Gamma_{in} = S_{11}$$

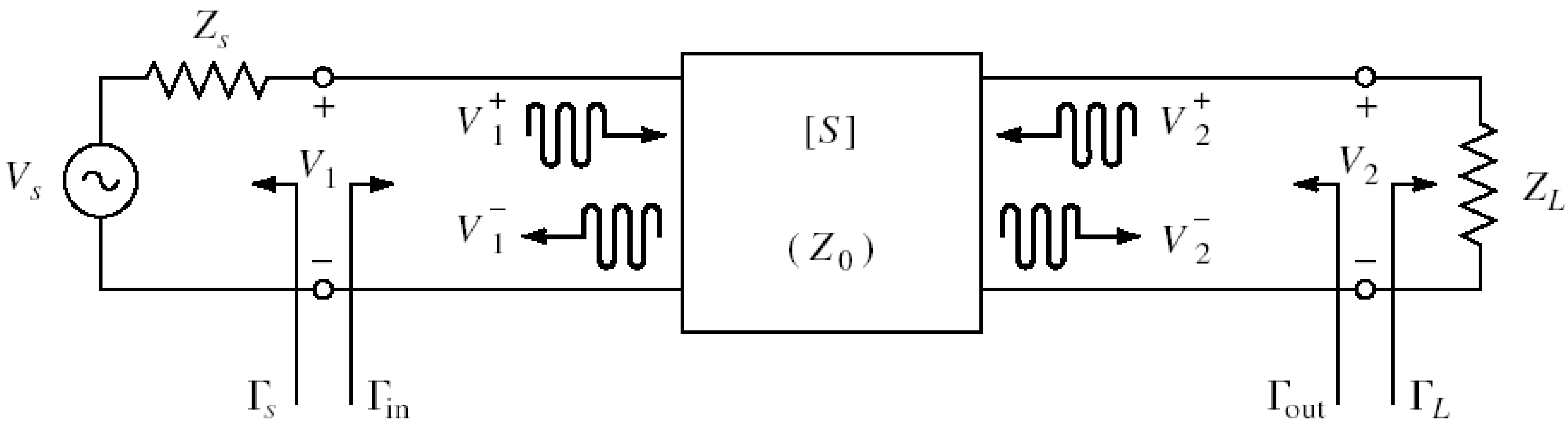
Input and output can be treated independently

Amplifier as two-port



- For an amplifier two-port we are interested in:
  - **stability**
  - power gain
  - noise (sometimes – small signals)
  - linearity (sometimes – large signals)

Amplifier as two-port



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  - **power gain**
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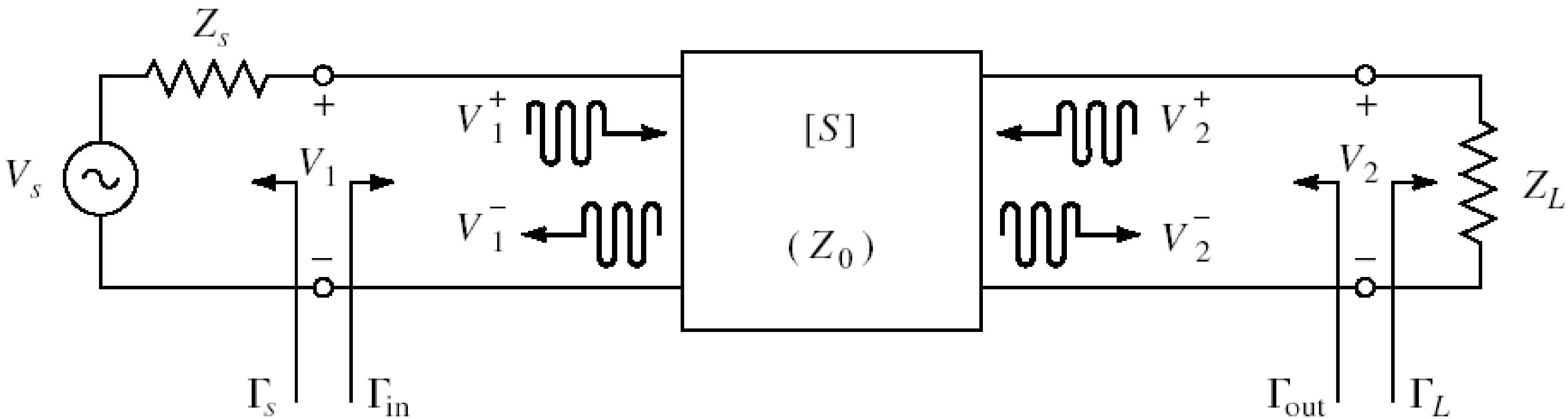
Amplifier as two-port



- For an amplifier two-port we are interested in:
  - stability
  - power gain
  - **noise** (sometimes – **small signals**)
  - linearity (sometimes – large signals)

Microwave Filters

Filter synthesis

- Filter is designed with lumped elements (L/C) followed by implementation with distributed elements (transmission lines)
  - general
  - analytical relationships easy to implement on the computer
  - efficient
- The preferred procedure is **insertion loss method**

Insertion loss method

$$P_{LR} = \frac{P_S}{P_L} = \frac{1}{1 - |\Gamma(\omega)|^2}$$

- $|\Gamma(\omega)|^2$ is an even function of ω

$$|\Gamma(\omega)|^2 = \frac{M(\omega^2)}{M(\omega^2) + N(\omega^2)}$$

$$P_{LR} = 1 + \frac{M(\omega^2)}{N(\omega^2)}$$

- Choosing M and N polynomials appropriately leads to a filter with a completely specified frequency response

Insertion loss method

- We control the power loss ratio/attenuation introduced by the filter:
  - in the passband (pass all frequencies)
  - in the stopband (reject all frequencies)

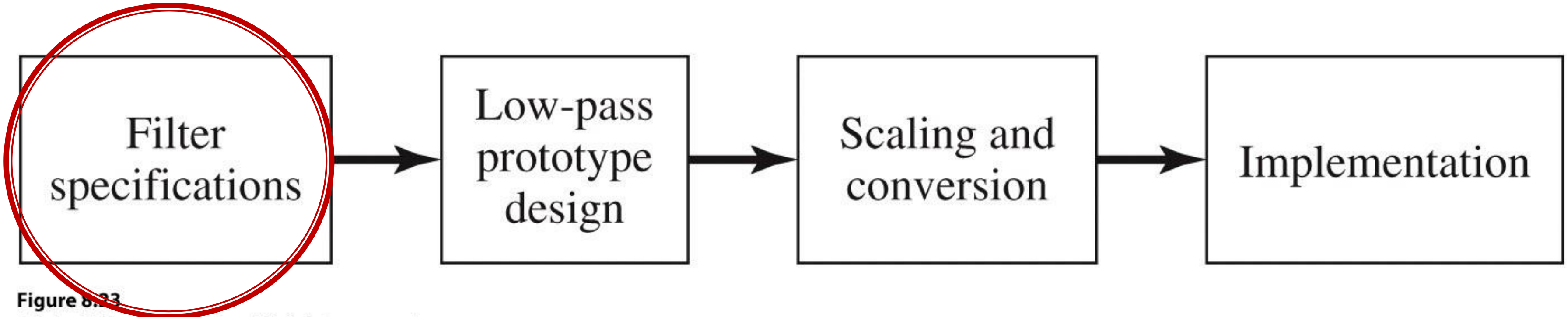
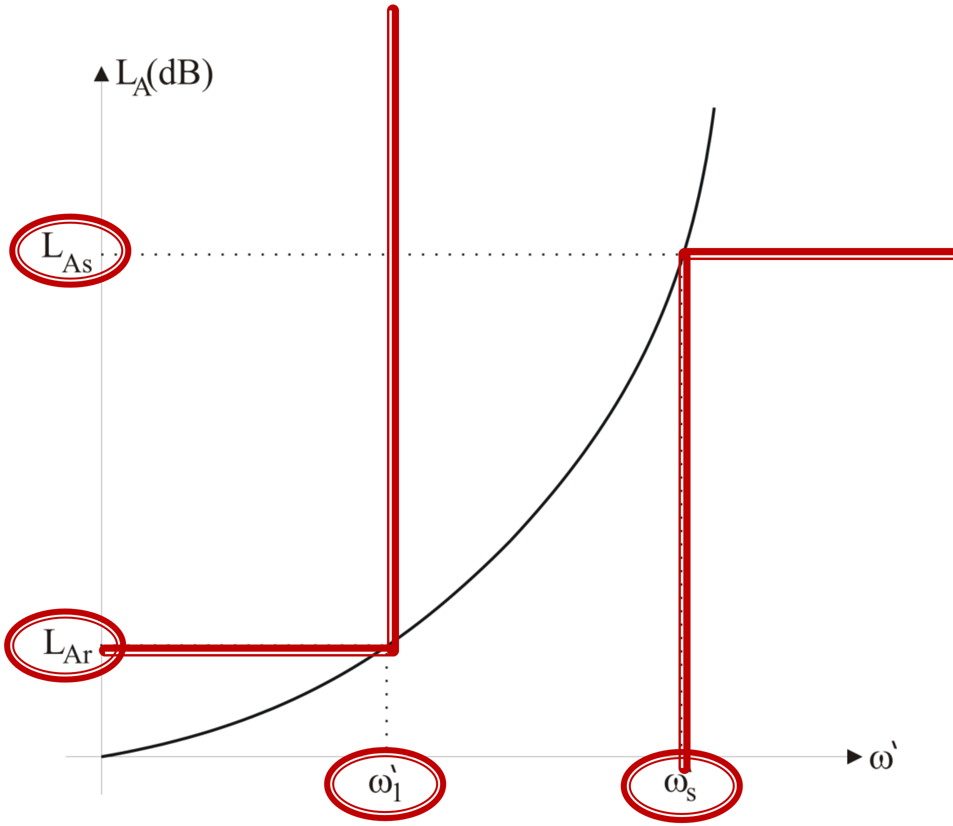


Figure 8.23

Filter specifications

- Attenuation
  - in passband
  - in stopband
  - most often in **dB**
- Frequency range
  - passband
  - stopband
  - cutoff frequency ω_1' usually normalized (**= 1**)



Insertion loss method

- We choose the right polynomials to design an **low-pass** filter (prototype)
- The low-pass prototype are then converted to the desired other types of filters
  - low-pass, high-pass, bandpass, or bandstop

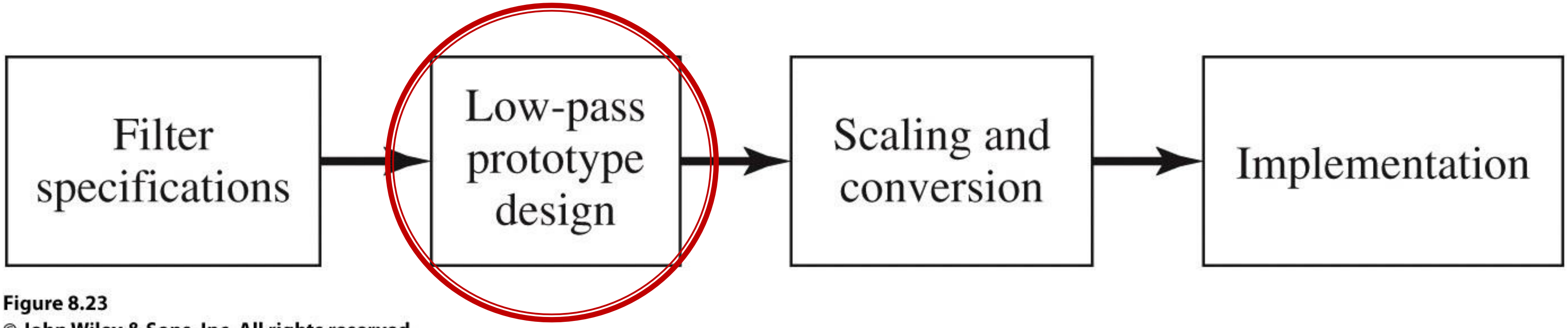


Figure 8.23

Maximally Flat/Equal ripple LPF Prototype

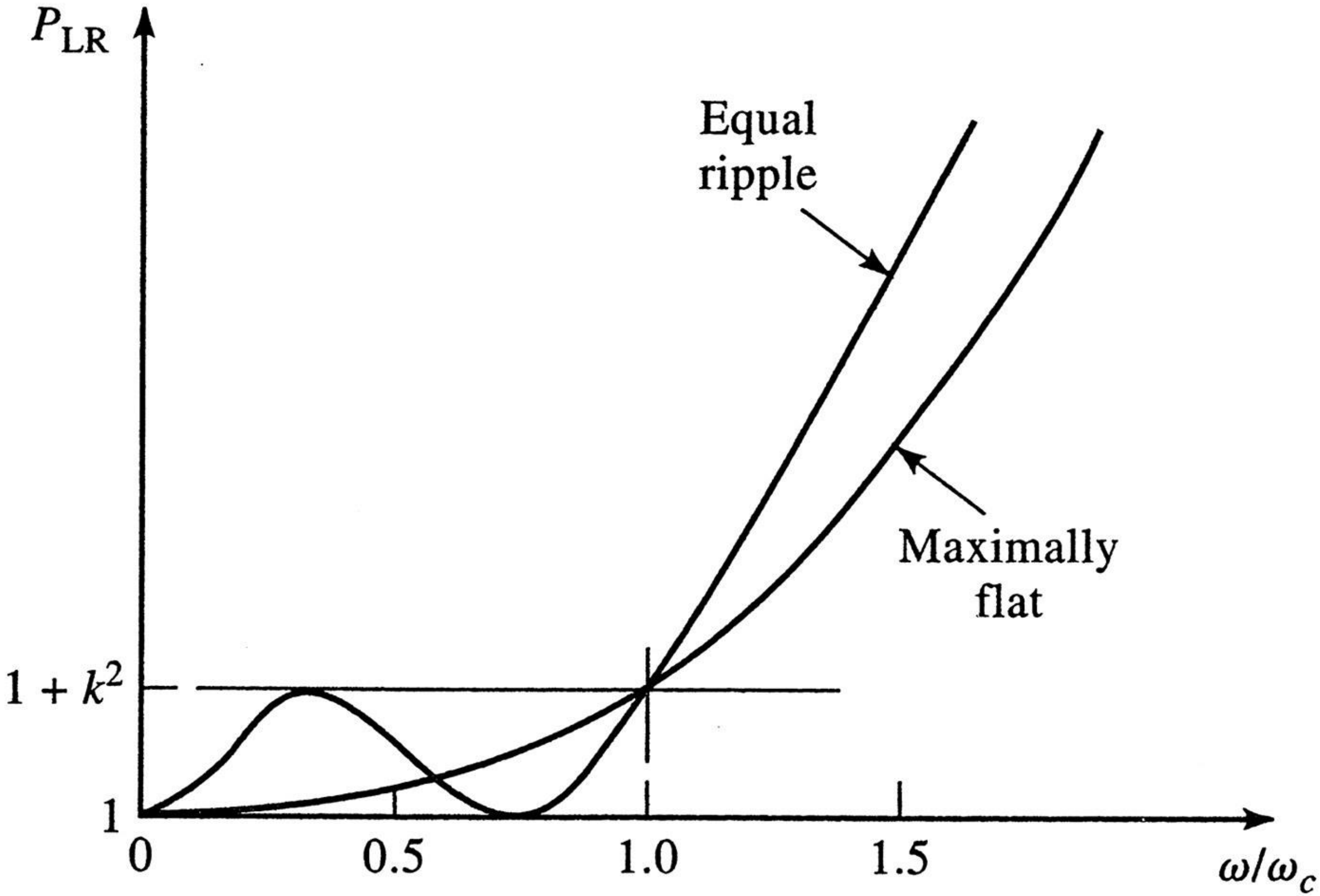
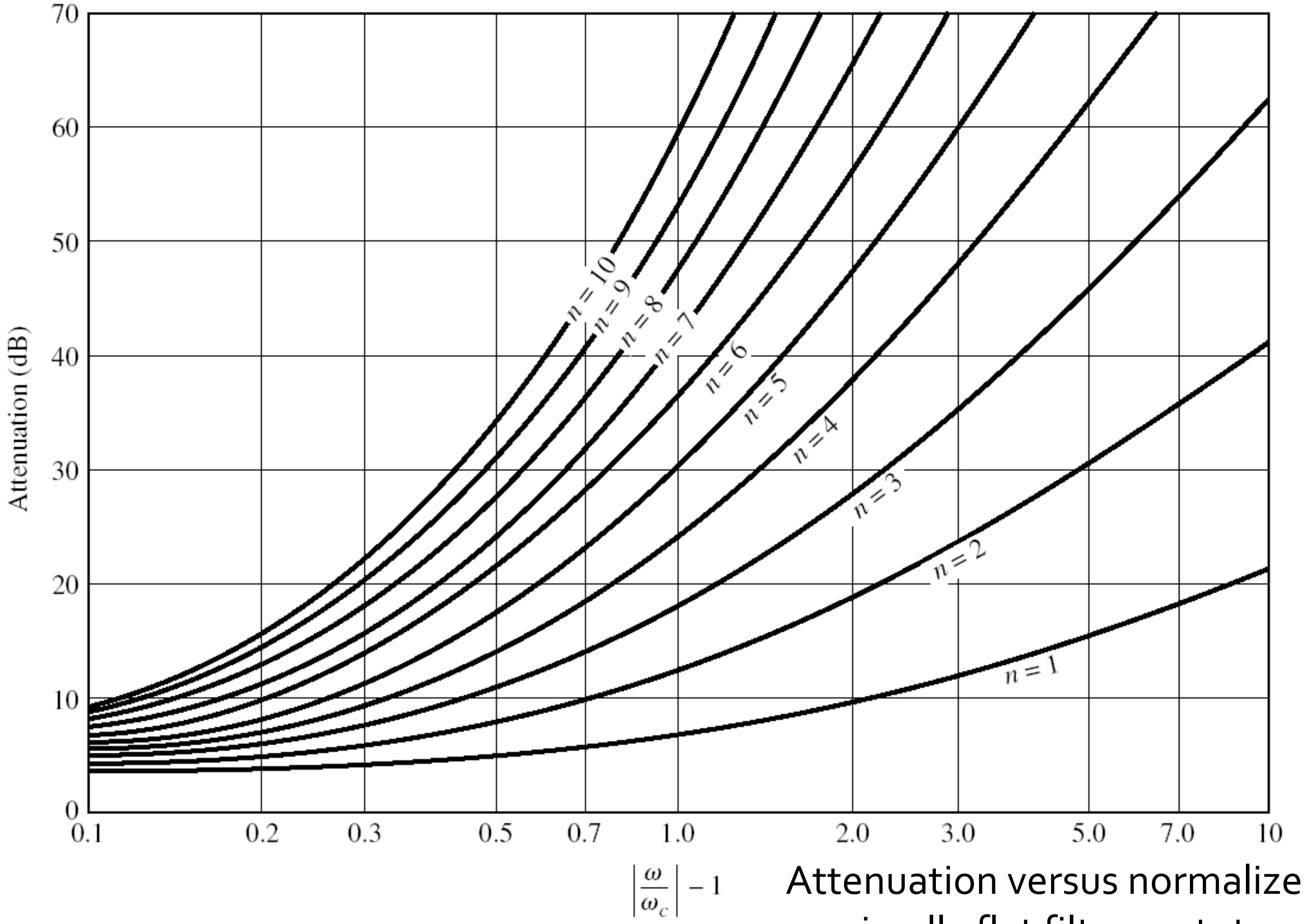


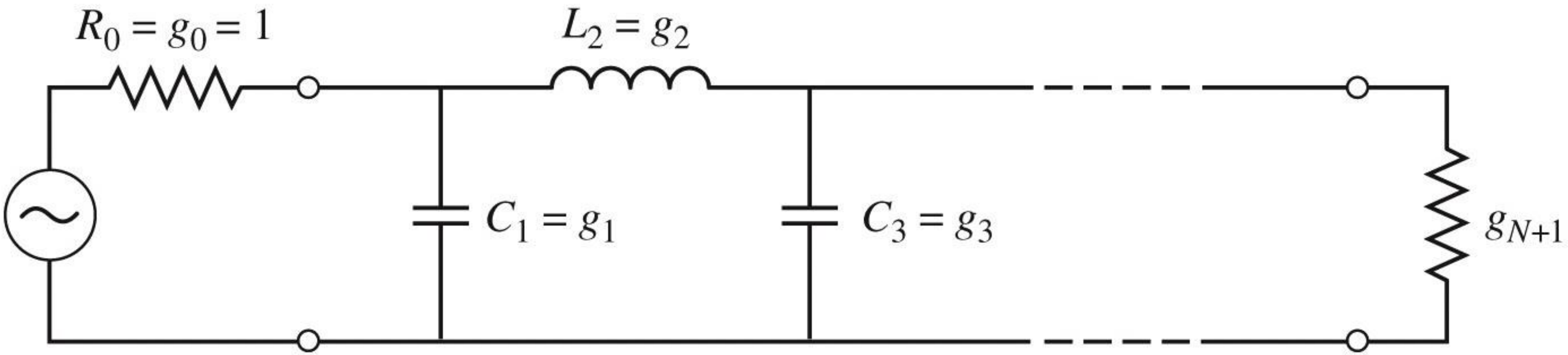
Figure 8.21

Maximally flat filter prototypes

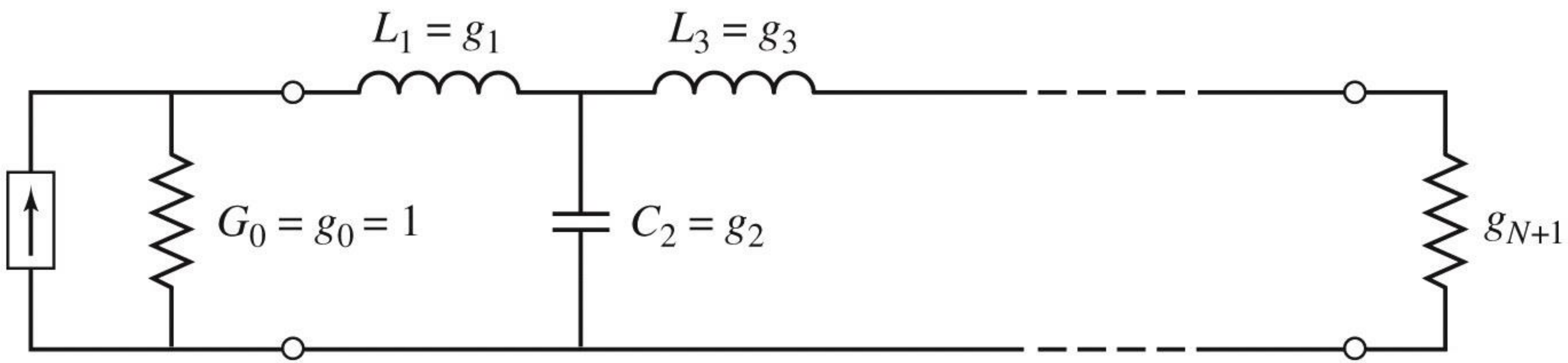


Attenuation versus normalized frequency for maximally flat filter prototypes

Prototype Filters



(a)



(b)

Maximally Flat LPF Prototype

- Formulas for filter parameters

$$g_0 = 1$$

$$g_k = 2 \cdot \sin\left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot N}\right] \quad , \quad k = 1, N$$

$$g_{N+1} = 1$$

Maximally Flat LPF Prototype

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

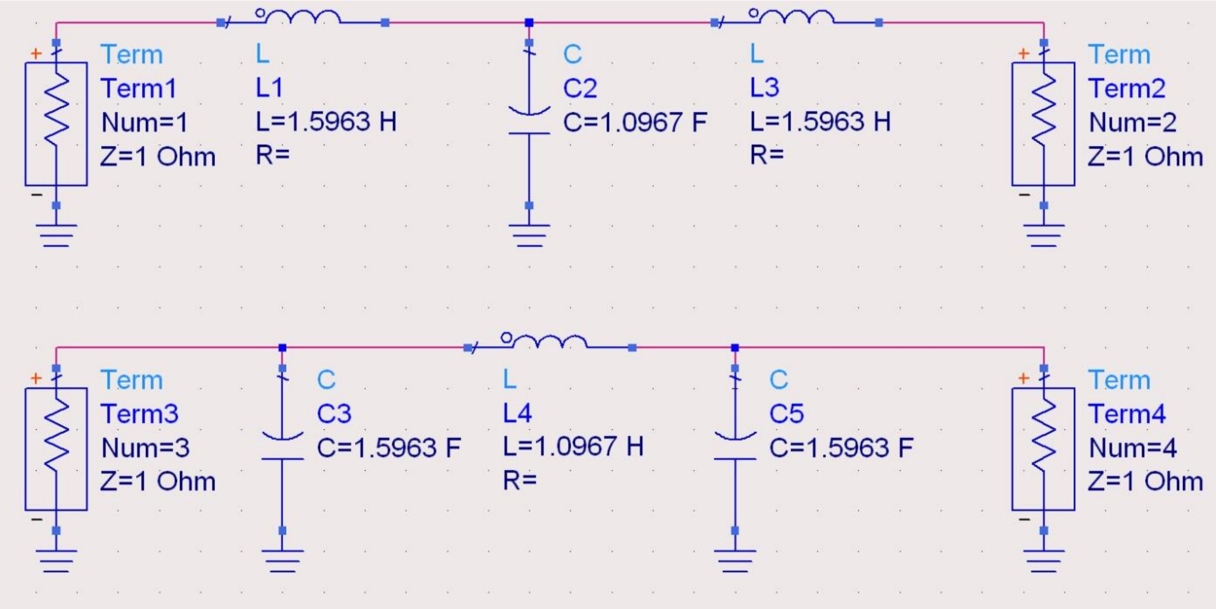
Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

Table 8.3

Continue

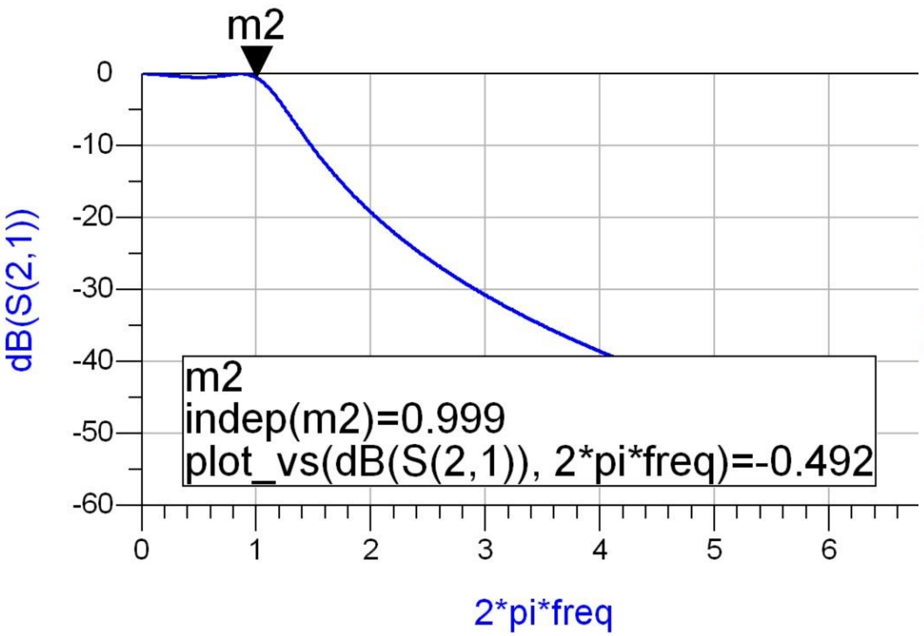
LPF Prototype

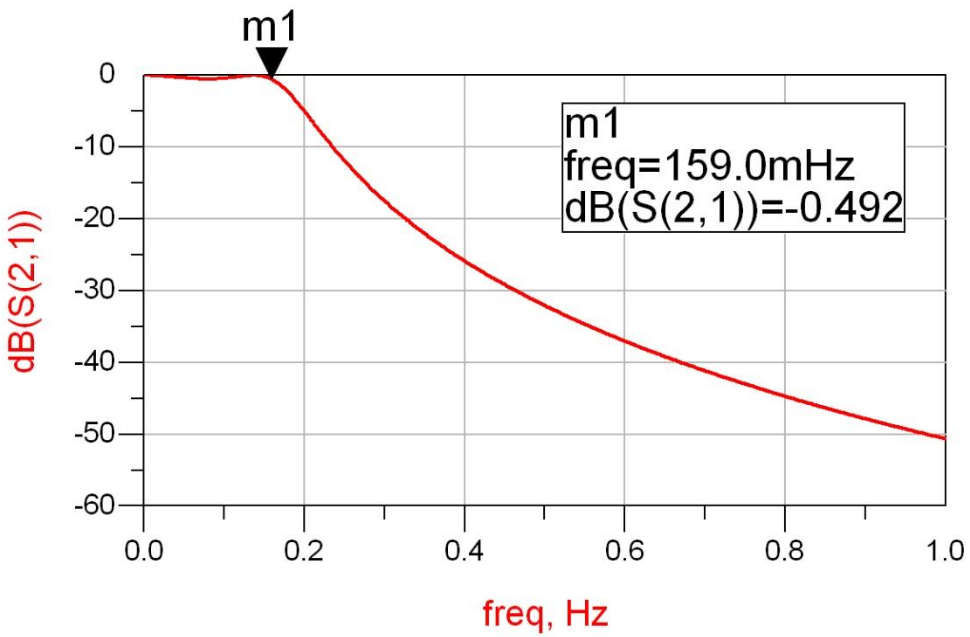
- 0.5dB equal-ripple table or design formulas:
  - g1 = 1.5963 = L1/C3,
  - g2 = 1.0967 = C2/L4,
  - g3 = 1.5963 = L3/C5,
  - g4=1.000 =R_L



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Impedance and Frequency Scaling

- After computing prototype filter's elements:
  - Low-Pass Filters (**LPF**)
  - cutoff frequency **ω_o = 1 rad/s** (f_o = 0.159 Hz)
  - connected to a source with **R = 1Ω**
- component values can be scaled in terms of impedance and frequency

Impedance and Frequency Scaling

- LPF Prototype is only used as an intermediate step
  - Low-Pass Filter (LPF)
  - cutoff frequency ω_o = 1 rad/s (f_o = 0.159 Hz)
  - connected to a source with R = 1Ω

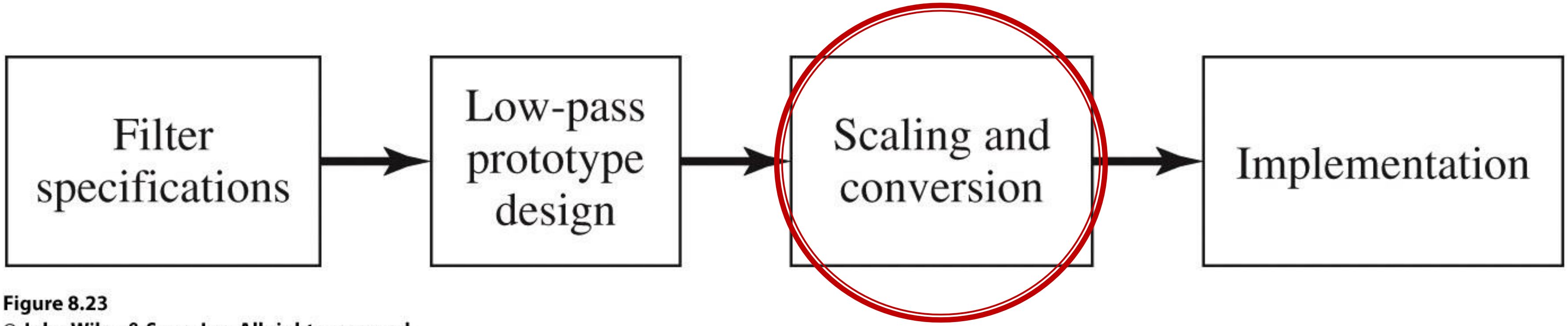


Figure 8.23

Impedance Scaling

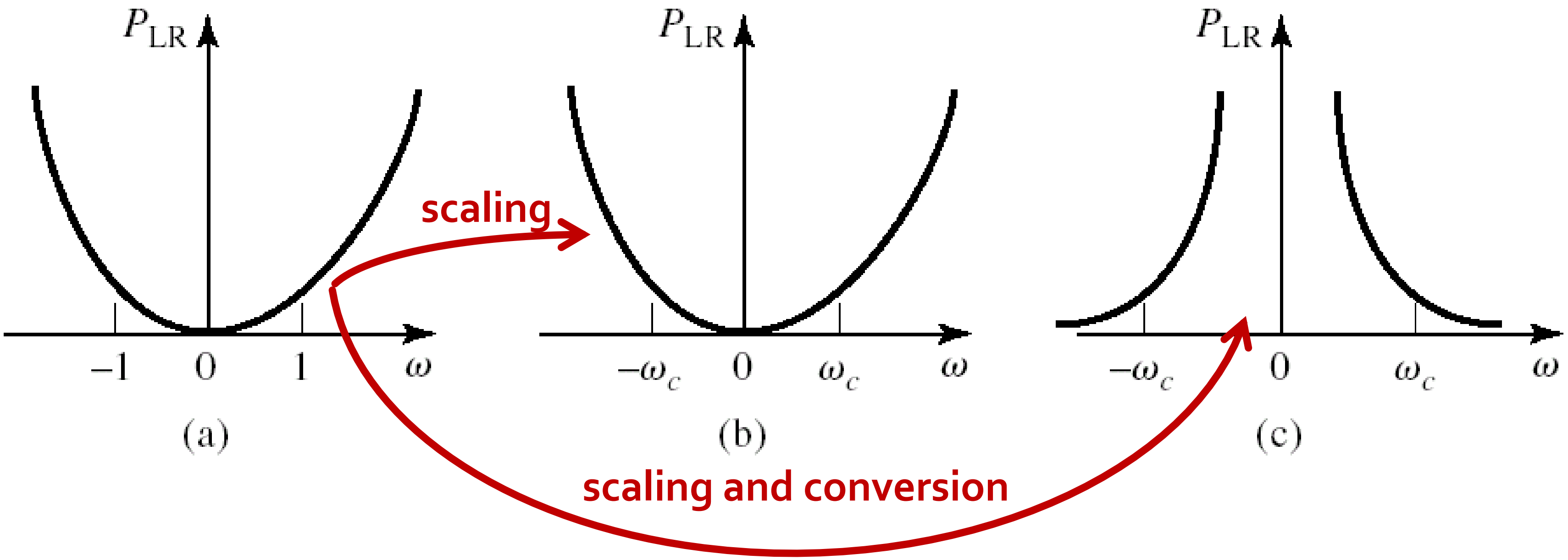
- To design a filter which will work with a source resistance of R_o we multiplying all the impedances of the prototype design by R_o ("'" denotes scaled values)

$$R_s' = R_0 \cdot (R_s = 1) \qquad R_L' = R_0 \cdot R_L$$

$$L' = R_0 \cdot L \qquad C' = \frac{C}{R_0}$$

Frequency Scaling

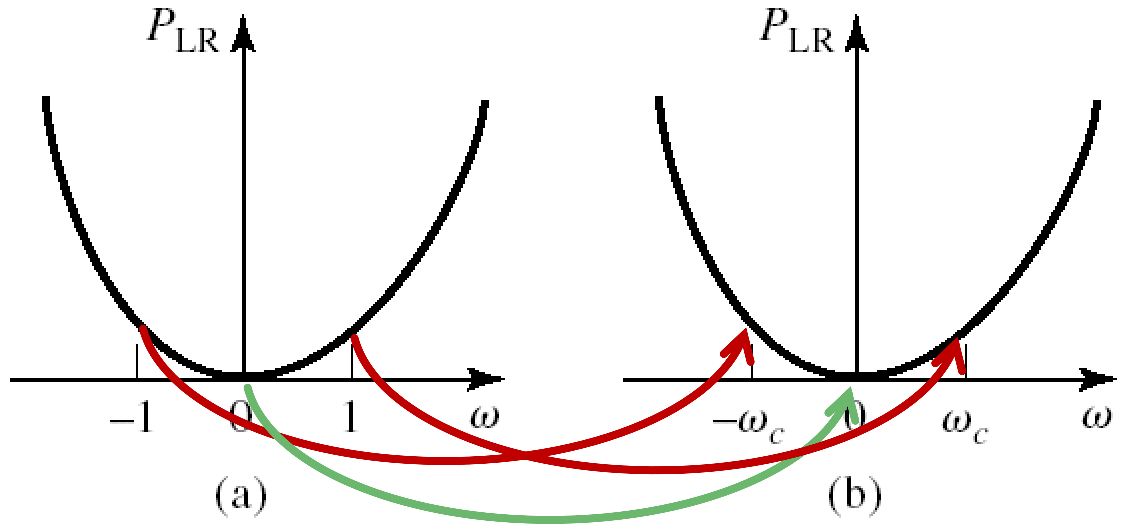
- changing the cutoff frequency – (fig. b)
- changing the type (for example LPF → HPF – fig. c) requires also conversion



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype from unity to ω_c we apply a variable substitution

$$\omega \leftarrow \frac{\omega}{\omega_c}$$



Frequency Scaling

- To change the cutoff frequency of a low-pass prototype we apply a variable substitution:

$$\omega \leftarrow \frac{\omega}{\omega_c}$$

- Equivalent to the widening of the power loss filter response

$$P'_{LR}(\omega) = P_{LR}\left(\frac{\omega}{\omega_c}\right)$$

$$j \cdot X_k = j \cdot \frac{\omega}{\omega_c} \cdot L_k = j \cdot \omega \cdot L'_k \qquad j \cdot B_k = j \cdot \frac{\omega}{\omega_c} \cdot C_k = j \cdot \omega \cdot C'_k$$

Frequency Scaling LPF → LPF

- New element values for frequency scaling:

$$L'_k = \frac{L_k}{\omega_c} \qquad C'_k = \frac{C_k}{\omega_c}$$

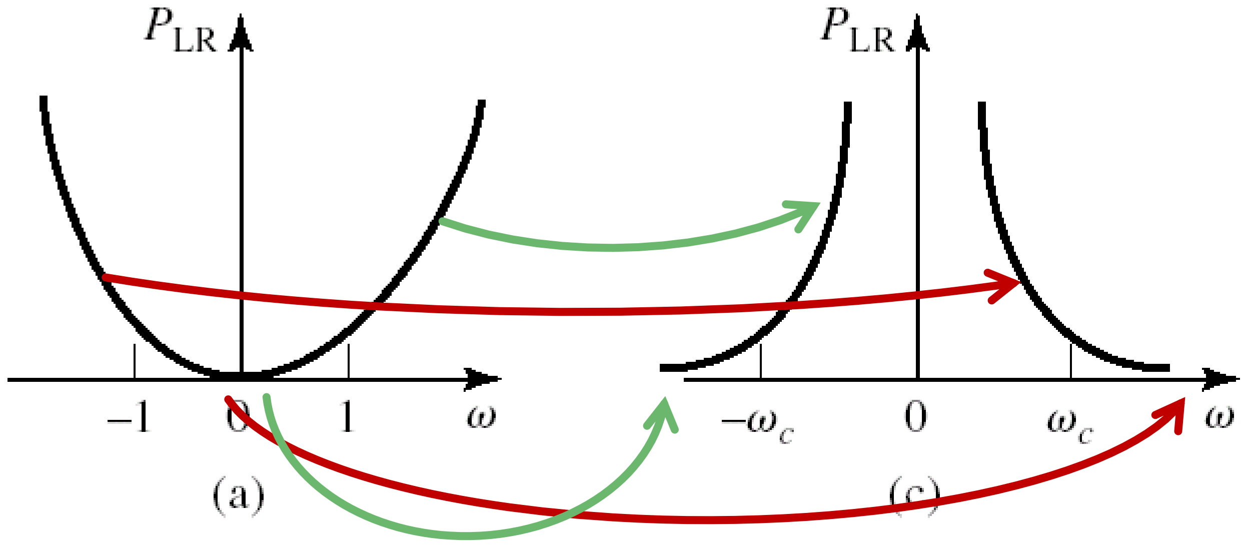
- When both impedance and frequency scaling are required:

$$L'_k = \frac{R_0 \cdot L_k}{\omega_c} \qquad C'_k = \frac{C_k}{R_0 \cdot \omega_c}$$

Low-pass to high-pass transformation LPF → HPF

- Variable substitution for LPF → HPF:

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$



High-pass transformation LPF → HPF

- Variable substitution for LPF → HPF :

$$\omega \leftarrow -\frac{\omega_c}{\omega}$$

$$j \cdot X_k = -j \cdot \frac{\omega_c}{\omega} \cdot L_k = \frac{1}{j \cdot \omega \cdot C_k'} \qquad j \cdot B_k = -j \cdot \frac{\omega_c}{\omega} \cdot C_k = \frac{1}{j \cdot \omega \cdot L_k'}$$

- Impedance scaling can be included

$$C_k' = \frac{1}{R_0 \cdot \omega_c \cdot L_k} \qquad L_k' = \frac{R_0}{\omega_c \cdot C_k}$$

- In the schematic series **inductors** must be replaced with series **capacitors**, and shunt **capacitors** must be replaced with shunt **inductors**

Bandpass Transformation LPF → BPF

- Variable substitution for LPF → BPF:

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

- where we use the fractional bandwidth of the passband and the center frequency

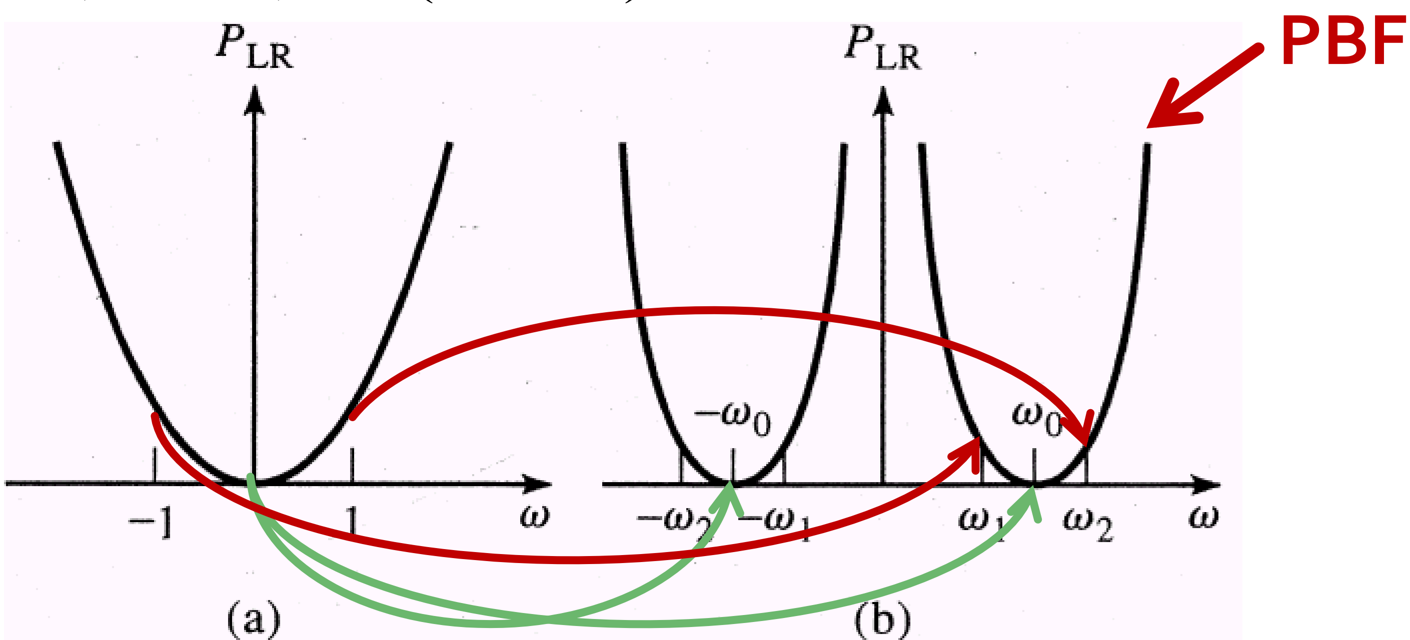
$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0} \qquad \omega_0 = \sqrt{\omega_1 \cdot \omega_2}$$

Bandpass Transformation LPF → BPF

$$\omega = \omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega_0}{\omega_0} - \frac{\omega_0}{\omega_0}\right) = 0 \qquad \omega = -\omega_0 \rightarrow \frac{1}{\Delta}\left(\frac{-\omega_0}{\omega_0} - \frac{\omega_0}{-\omega_0}\right) = 0$$

$$\omega = \omega_1 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega_1^2 - \omega_0^2}{\omega_0 \cdot \omega_1}\right) = -1$$

$$\omega = \omega_2 \rightarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega_2^2 - \omega_0^2}{\omega_0 \cdot \omega_2}\right) = 1$$



Bandpass Transformation LPF → BPF

$$j \cdot X_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cdot L_k = j \cdot \frac{\omega \cdot L_k}{\Delta \cdot \omega_0} - j \cdot \frac{\omega_0 \cdot L_k}{\Delta \cdot \omega} = j \cdot \omega \cdot L_k' - j \frac{1}{\omega \cdot C_k'}$$

$$j \cdot B_k = \frac{j}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \cdot C_k = j \cdot \frac{\omega \cdot C_k}{\Delta \cdot \omega_0} - j \cdot \frac{\omega_0 \cdot C_k}{\Delta \cdot \omega} = j \cdot \omega \cdot C_k' - j \frac{1}{\omega \cdot L_k'}$$

- A series **inductor** in the prototype filter is transformed to a **series LC** circuit **in series**

$$L_k' = \frac{L_k}{\Delta \cdot \omega_0} \qquad C_k' = \frac{\Delta}{\omega_0 \cdot L_k}$$

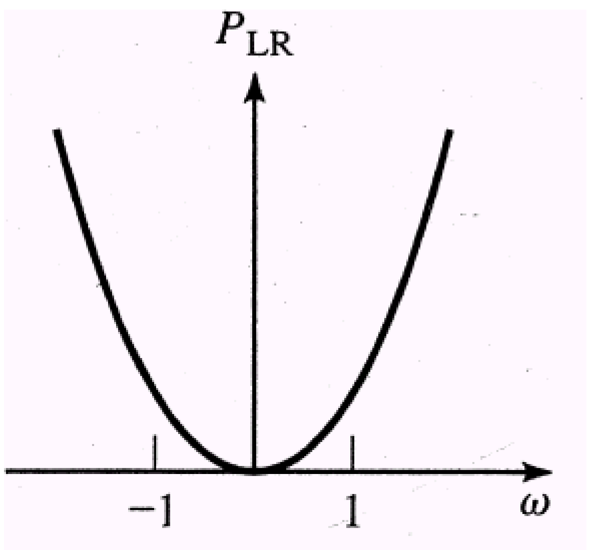
- A shunt **capacitor** in the prototype filter is transformed to a **shunt LC** circuit **in parallel**

$$L_k' = \frac{\Delta}{C_k \cdot \omega_0} \qquad C_k' = \frac{C_k}{\omega_0 \cdot \Delta}$$

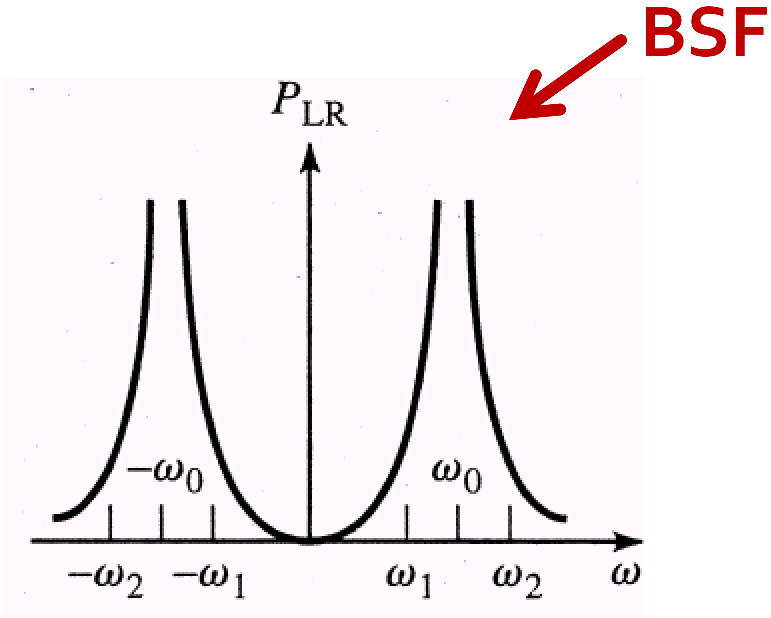
Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1}$$

$$\omega = \omega_0 \rightarrow \frac{-\Delta}{\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)} = \frac{-\Delta}{\left(\frac{\omega_0}{\omega_0} - \frac{\omega_0}{\omega_0} \right)} \rightarrow \pm\infty$$



(a)



(c)

Bandstop Transformation LPF → BSF

$$\omega \leftarrow -\Delta \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)^{-1}$$

- A series **inductor** in the prototype filter is transformed to a **shunt LC** circuit **in series**

$$L'_k = \frac{\Delta \cdot L_k}{\omega_0} \qquad C'_k = \frac{1}{\omega_0 \cdot \Delta \cdot L_k}$$

- A shunt **capacitor** in the prototype filter is transformed to a **series LC** circuit **in parallel**

$$L'_k = \frac{1}{\Delta \cdot \omega_0 \cdot C_k} \qquad C'_k = \frac{\Delta \cdot C_k}{\omega_0}$$

Summary of Prototype Filter Transformations

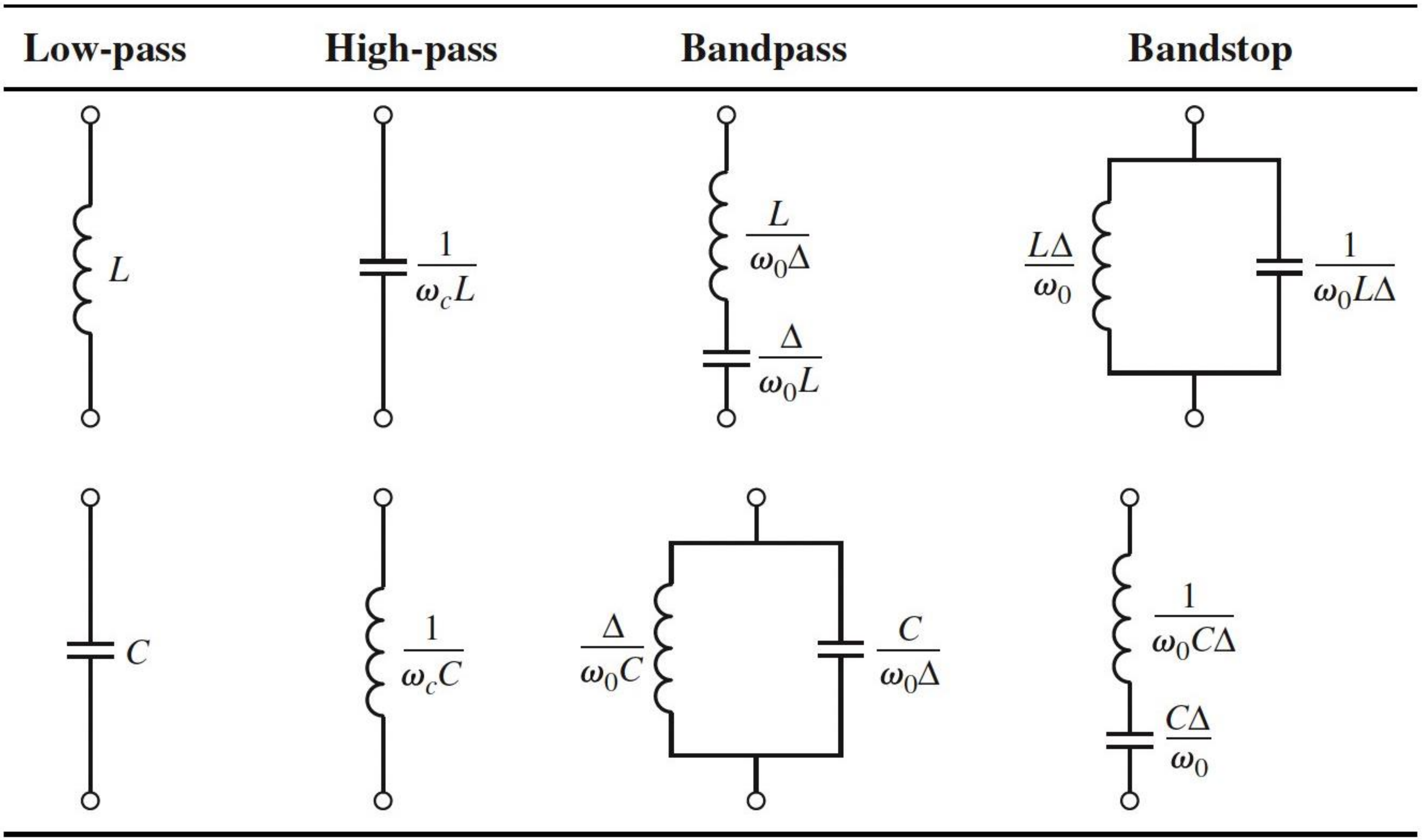


Table 8.6

Example

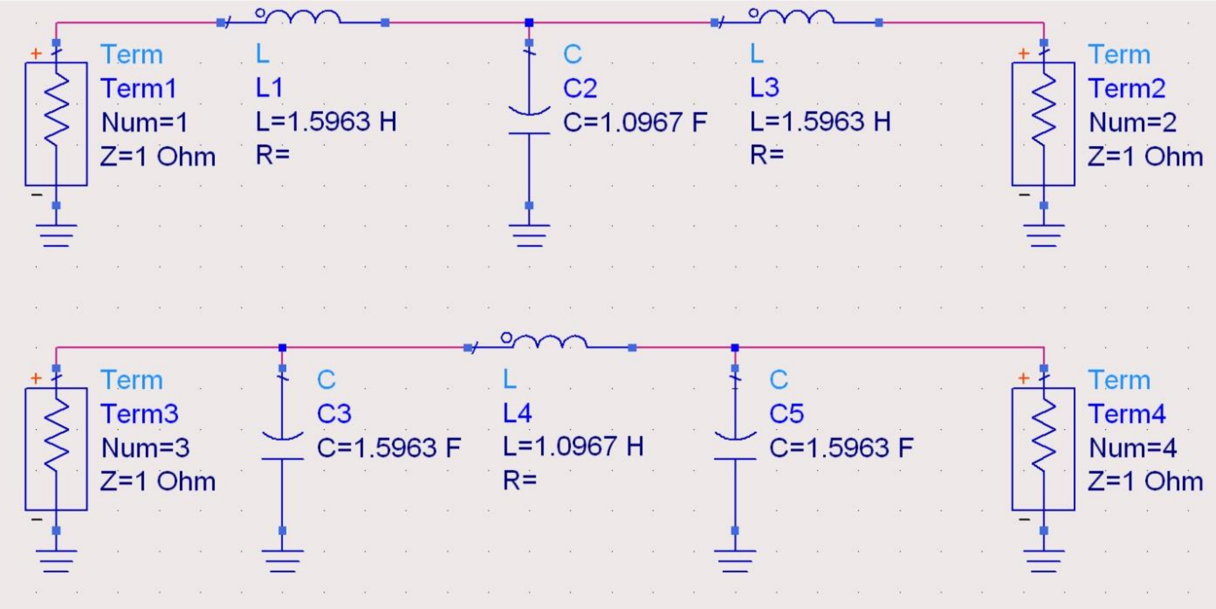
- Design a 3rd order **bandpass** filter with 0.5 dB ripples in passband. The **center frequency** of the filter should be 1 GHz. The **fractional bandwidth** of the passband should be 10%, and the **impedance** 50Ω.

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 \, rad/s$$

$$\Delta = 0.1$$

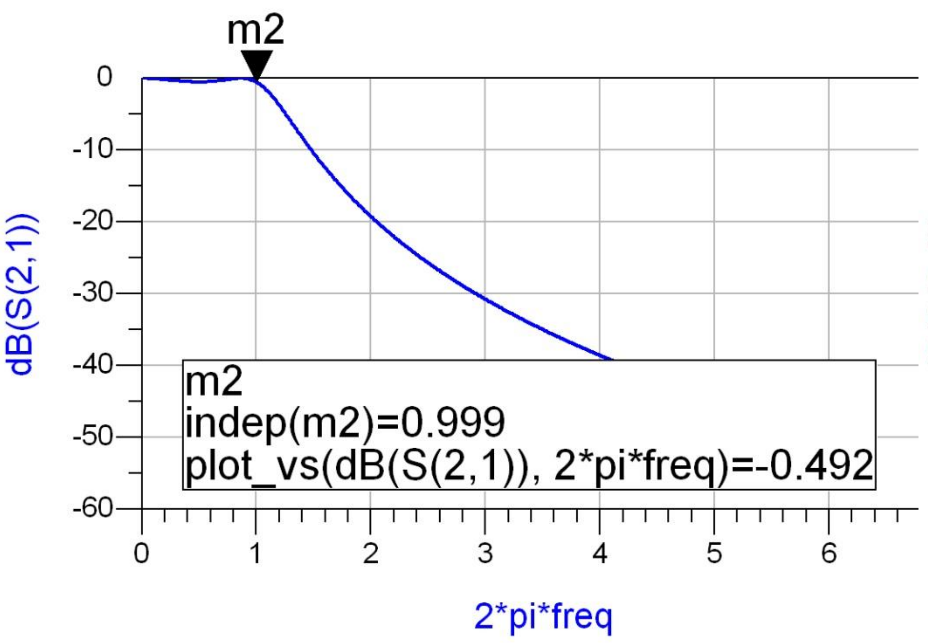
LPF Prototype

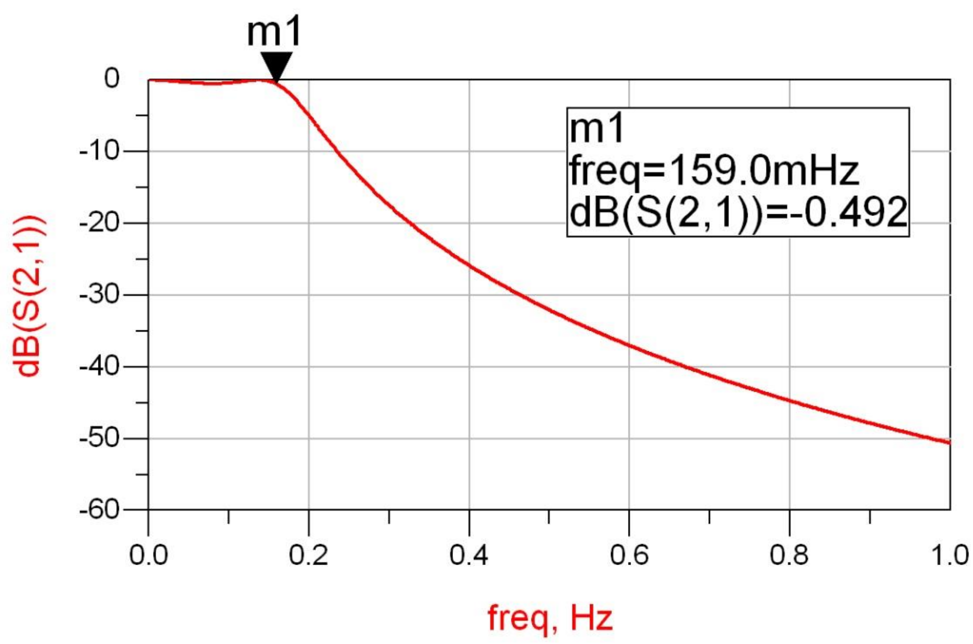
- 0.5dB equal-ripple table or design formulas:
  - g1 = 1.5963 = L1/C3,
  - g2 **=** 1.0967 = C2/L4,
  - g3 = 1.5963 = L3/C5,
  - g4=1.000 =R_L



LPF Prototype

- ω_o = 1 rad/s ($f_o = \omega_o / 2\pi$ = 0.159 Hz)





Bandpass Transformation / BPF

$$\omega_0 = 2 \cdot \pi \cdot 1GHz = 6.283 \cdot 10^9 \, rad/s \qquad \Delta = \frac{\Delta\omega}{\omega_0} = \frac{\Delta f}{f_0} = 0.1 \qquad R_0 = 50\,\Omega$$

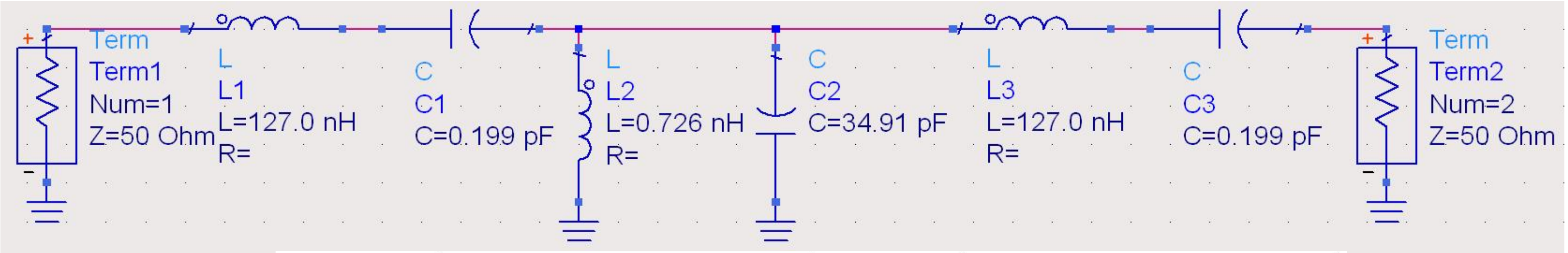
g1 = 1.5963 = L1,
g2 = 1.0967 = C2,
g3 = 1.5963 = L3,
g4=1.000 =R_L

$$L_1' = \frac{L_1 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\,nH \qquad C_1' = \frac{\Delta}{\omega_0 \cdot L_1 \cdot R_0} = 0.199\,pF$$

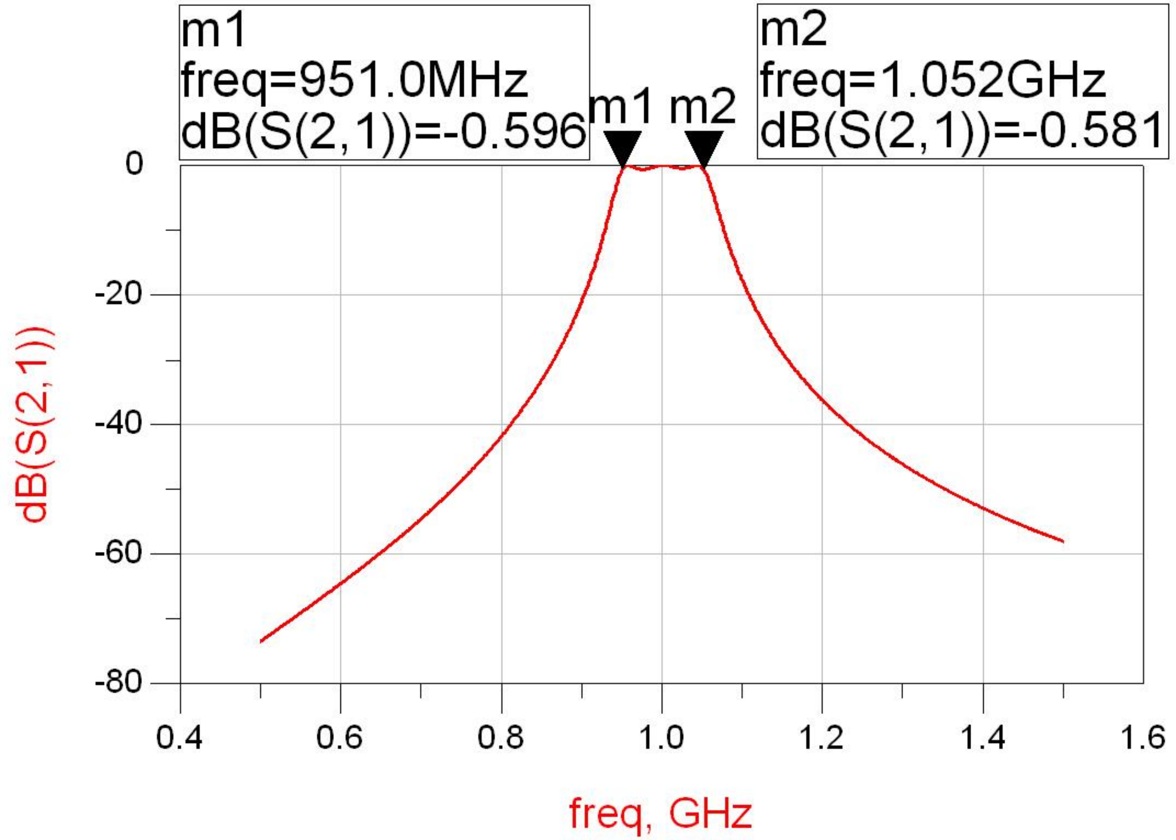
$$L_2' = \frac{\Delta \cdot R_0}{\omega_0 \cdot C_2} = 0.726\,nH \qquad C_2' = \frac{C_2}{\Delta \cdot \omega_0 \cdot R_0} = 34.91\,pF$$

$$L_3' = \frac{L_3 \cdot R_0}{\Delta \cdot \omega_0} = 127.0\,nH \qquad C_3' = \frac{\Delta}{\omega_0 \cdot L_3 \cdot R_0} = 0.199\,pF$$

ADS



Term
Term1
Num=1
Z=50 Ohm
L
L1
L=127.0 nH
R=
C
C1
C=0.199 pF
L
L2
L=0.726 nH
R=
C
C2
C=34.91 pF
L
L3
L=127.0 nH
R=
C
C3
C=0.199 pF
Term
Term2
Num=2
Z=50 Ohm



m1
freq=951.0MHz
dB(S(2,1))=-0.596
m2
freq=1.052GHz
dB(S(2,1))=-0.581
m1 m2
0
-20
-40
-60
-80
dB(S(2,1))
0.4
0.6
0.8
1.0
1.2
1.4
1.6
freq, GHz

Microwave Filters Implementation

Microwave Filters Implementation

- The lumped-element (L, C) filter design generally works well **only** at low frequencies (RF):
  - lumped-element inductors and capacitors are generally available only for a limited range of values, and can be difficult to implement at microwave frequencies
  - difficulty to obtain the (very low) required tolerance for elements

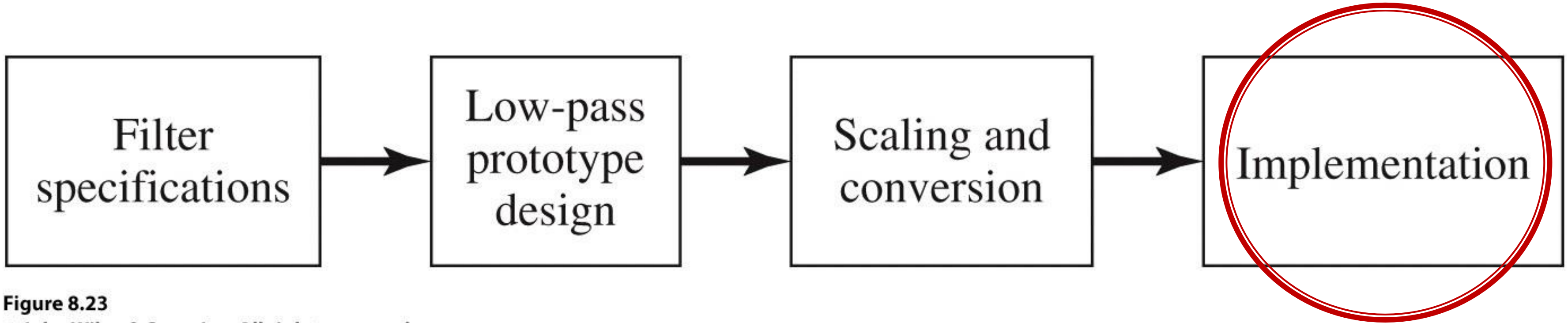


Figure 8.23

Richards' Transformation

- Impedance seen at the input of a line loaded with Z_L

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

- We prefer the load impedance to be:
  - open circuit ($Z_L = \infty$) $\quad Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l$
  - short circuit ($Z_L = 0$) $\quad Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l$
- Input impedance is:
  - capacitive $\quad Z_{in,oc} = j \cdot X_C = \frac{1}{j \cdot B_C} \quad Z_0 \leftrightarrow \frac{1}{C} \quad \tan \beta \cdot l \leftrightarrow \omega$
  - inductive $\quad Z_{in,sc} = j \cdot X_L \quad Z_0 \leftrightarrow L \quad \tan \beta \cdot l \leftrightarrow \omega$

Richards' Transformation

- Variable substitution

$$\Omega = \tan\beta \cdot l = \tan\left(\frac{\omega \cdot l}{v_p}\right)$$

- With this variable substitution we define:
  - reactance of an inductor

$$j \cdot X_L = j \cdot \Omega \cdot L = j \cdot L \cdot \tan\beta \cdot l$$

  - susceptance of a capacitor

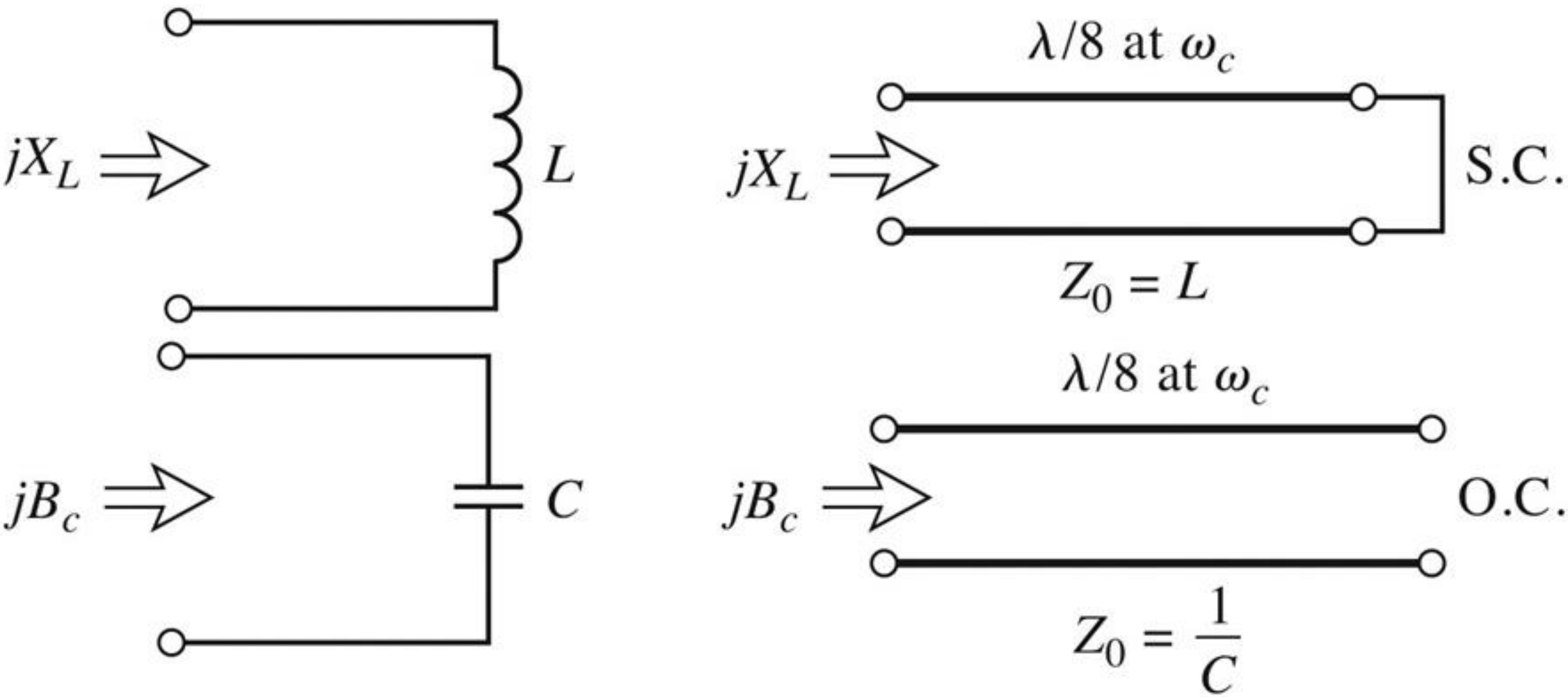
$$j \cdot B_C = j \cdot \Omega \cdot C = j \cdot C \cdot \tan\beta \cdot l$$

- The equivalent filter in Ω has a cutoff frequency at:

$$\Omega = 1 = \tan\beta \cdot l \quad \rightarrow \quad \beta \cdot l = \frac{\pi}{4} \quad \rightarrow \quad l = \frac{\lambda}{8}$$

Richards' Transformation

- allows implementation of the inductors and capacitors with lines **after** the transformation of the LPF prototype to the required type (LPF/HPF/BPF/BSF)



Richards' Transformation

- By choosing the open-circuited or short-circuited lines to be λ/8 at the desired cutoff frequency (ω_c) and the corresponding characteristic impedances (L/C from LPF prototype) we will obtain at frequencies around ω_c a behavior similar to that of the prototype filter.
  - At frequencies far from ω_c the behavior of the filter will no longer be identical to that of the prototype (in specific situations the correct behavior must be **verified**)
  - Frequency scaling is simplified: choosing the appropriate physical length of the line to have the electrical length λ/8 at the desired cutoff frequency
- All lines will have equal electrical lengths (λ/8) and thus comparable physical lengths, so the lines are called **commensurate** lines

Richards' Transformation

- At the frequency $\omega = 2 \cdot \omega_c$ the lines will be $\lambda/4$ long
$$l = \frac{\lambda}{4} \quad \Rightarrow \quad \beta \cdot l = \frac{\pi}{2} \quad \Rightarrow \quad \tan \beta \cdot l \to \infty$$
- an supplemental attenuation pole will occur at $2 \cdot \omega_c$ (LPF):
  - inductances (usually in series) $Z_{in,sc} = j \cdot Z_0 \cdot \tan \beta \cdot l \to \infty$
  - capacitances (usually shunt) $Z_{in,oc} = -j \cdot Z_0 \cdot \cot \beta \cdot l \to 0$

Richards' Transformation

- the periodicity of tan function implies the periodicity of the filter implemented with lines
  - the filter response will be repeated every $4 \cdot \omega_c$

$$\tan(\alpha + \pi) = \tan \alpha$$

$$\beta \cdot l \Big|_{\omega=\omega_c} = \frac{\pi}{4} \quad \Rightarrow \quad \frac{\omega_c \cdot l}{v_p} = \frac{\pi}{4} \quad \Rightarrow \quad \pi = \frac{(4 \cdot \omega_c) \cdot l}{v_p}$$

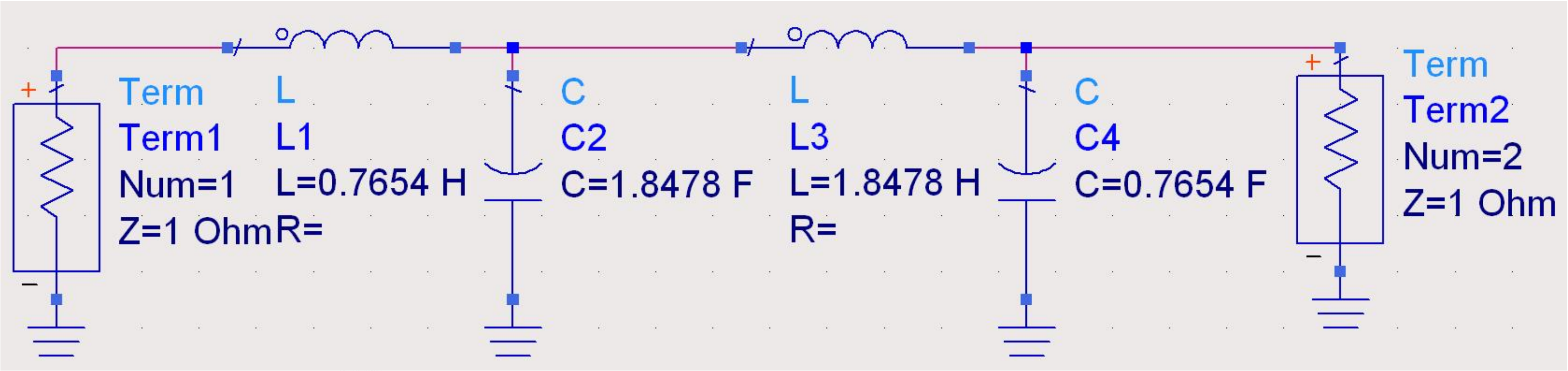
$$Z_{in}(\omega) = Z_{in}(\omega + 4 \cdot \omega_c) \quad \Rightarrow \quad P_{LR}(\omega) = P_{LR}(\omega + 4 \cdot \omega_c)$$

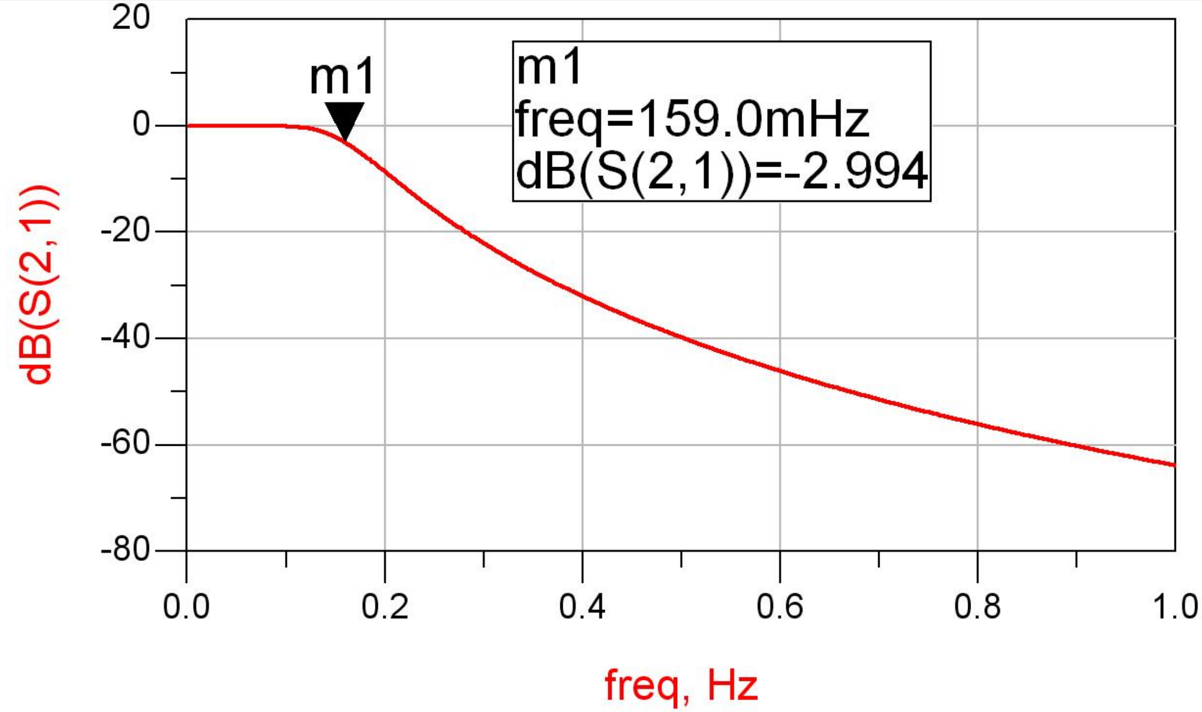
$$P_{LR}(4 \cdot \omega_c) = P_{LR}(0) \qquad P_{LR}(3 \cdot \omega_c) = P_{LR}(-\omega_c) \qquad P_{LR}(5 \cdot \omega_c) = P_{LR}(\omega_c)$$

Example

- Low-pass filter 4^{th} order, 4 GHz cutoff frequency, maximally flat design (working with 50Ω source and load)
- maximally flat table or formulas:
  - $g_1 = 0.7654 = L_1$
  - $g_2 = 1.8478 = C_2$
  - $g_3 = 1.8478 = L_3$
  - $g_4 = 0.7654 = C_4$
  - $g_5 = 1$ (**does not** need supplemental impedance matching – required only for even order equal-ripple filters)

LPF Prototype





Lumped elements

$\omega_c = 2 \cdot \pi \cdot 4GHz = 2.5133 \cdot 10^{10} rad/s$

g1 = 0.7654 = L1,
g2 = 1.8478 = C2,

g3 = 1.8478 = L3,
g4 = 0.7654 = C4,
g5 = 1 = RL

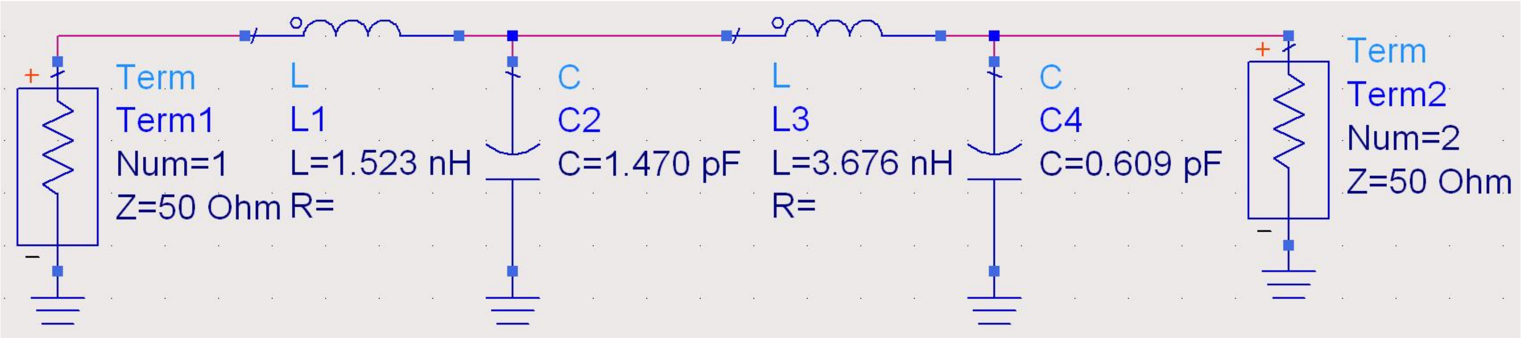
$$L_1' = \frac{R_0 \cdot L_1}{\omega_c} = 1.523nH$$

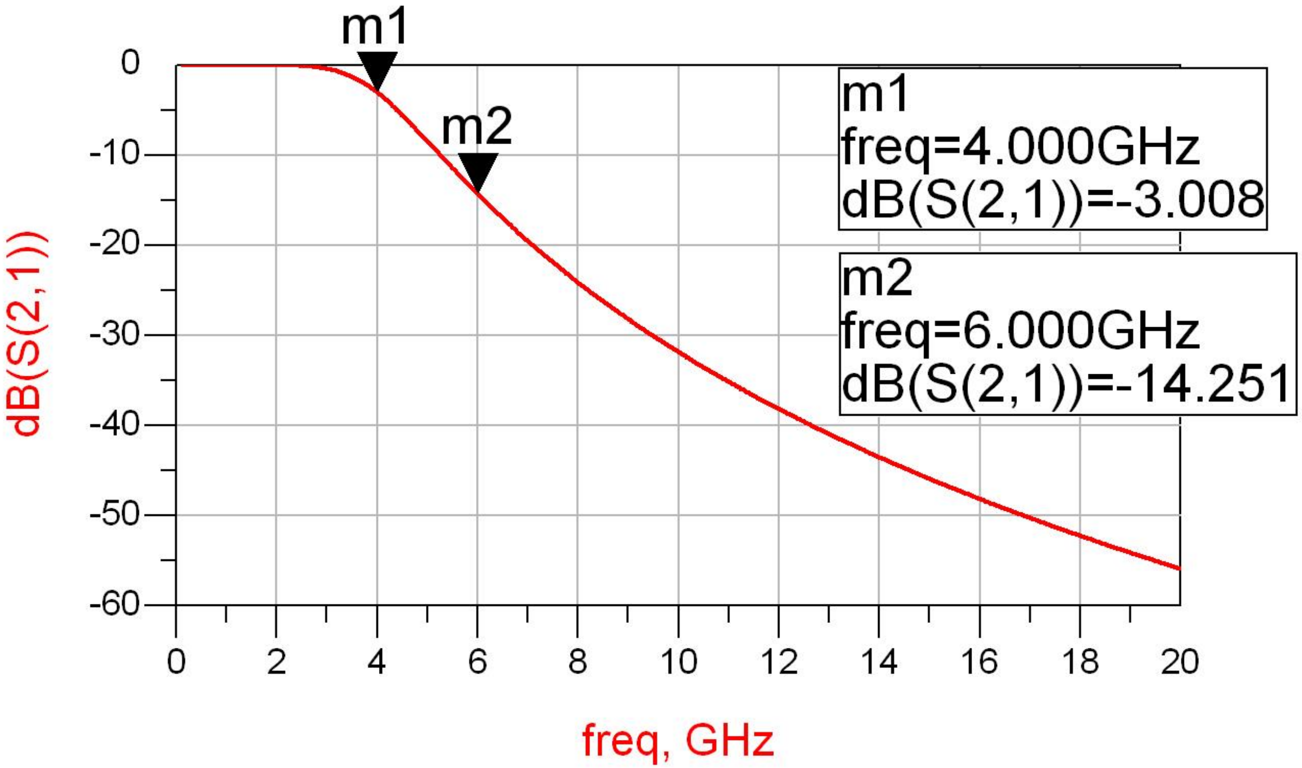
$$C_2' = \frac{C_2}{R_0 \cdot \omega_c} = 1.470pF$$

$$L_3' = \frac{R_0 \cdot L_3}{\omega_c} = 3.676nH$$

$$C_4' = \frac{C_4}{R_0 \cdot \omega_c} = 0.609pF$$

Lumped elements – ADS





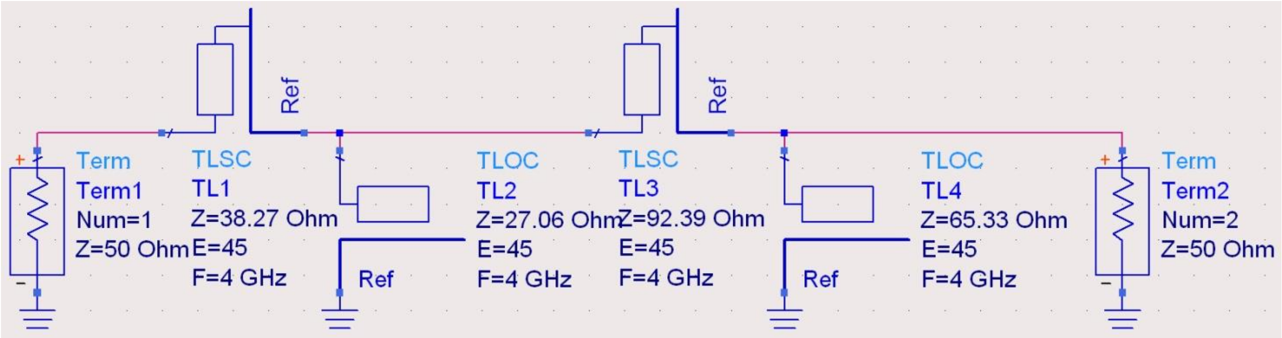
Richards' Transformation

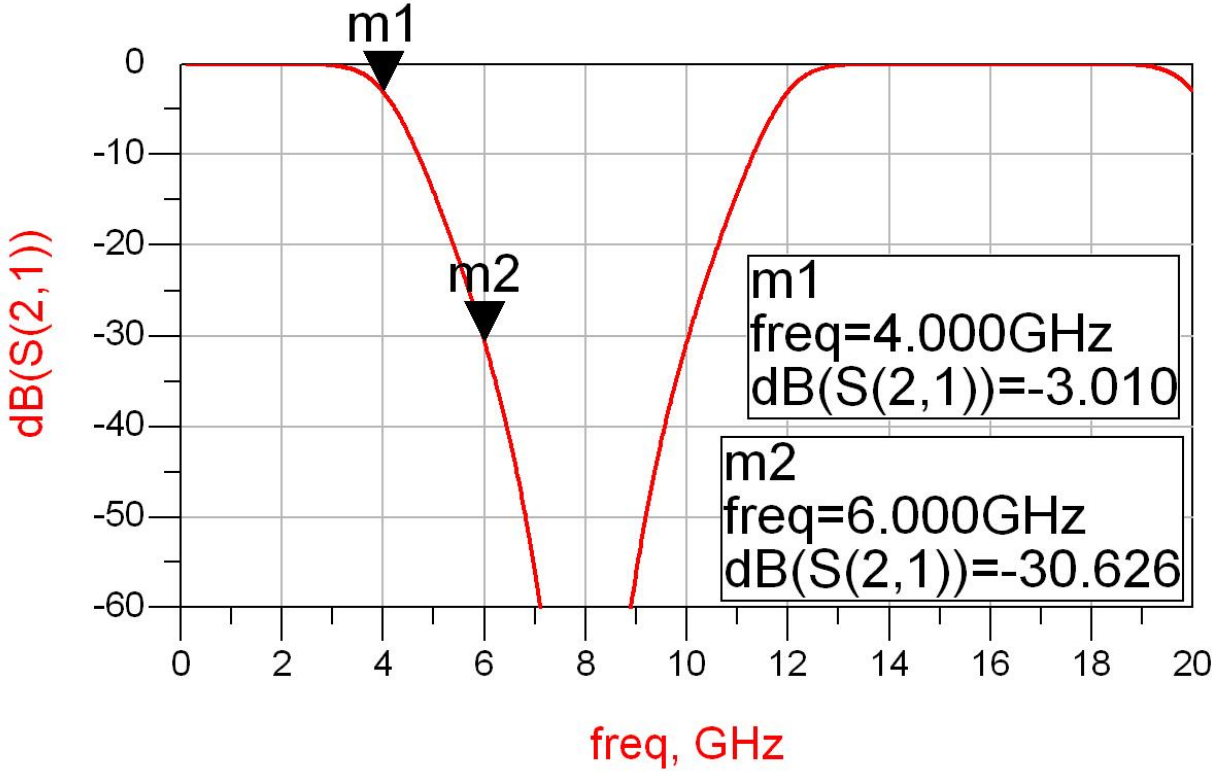
- LPF Prototype parameters:
  - g1 = 0.7654 = L1
  - g2 = 1.8478 = C2
  - g3 = 1.8478 = L3
  - g4 = 0.7654 = C4
- Normalized line impedances
  - z1 = 0.7654 = series / short circuit
  - z2 = 1 / 1.8478 = 0.5412 = shunt / open circuit
  - z3 = 1.8478 = series / short circuit
  - z4 = 1/ 0.7654 = 1.3065 = shunt / open circuit
- Impedance scaling by multiplying with Zo = 50Ω
- All lines must have the length equal to λ/8 (electrical length E = 45°) at 4GHz

$$Z_0 \leftrightarrow \frac{1}{C}$$

$$Z_0 \leftrightarrow L$$

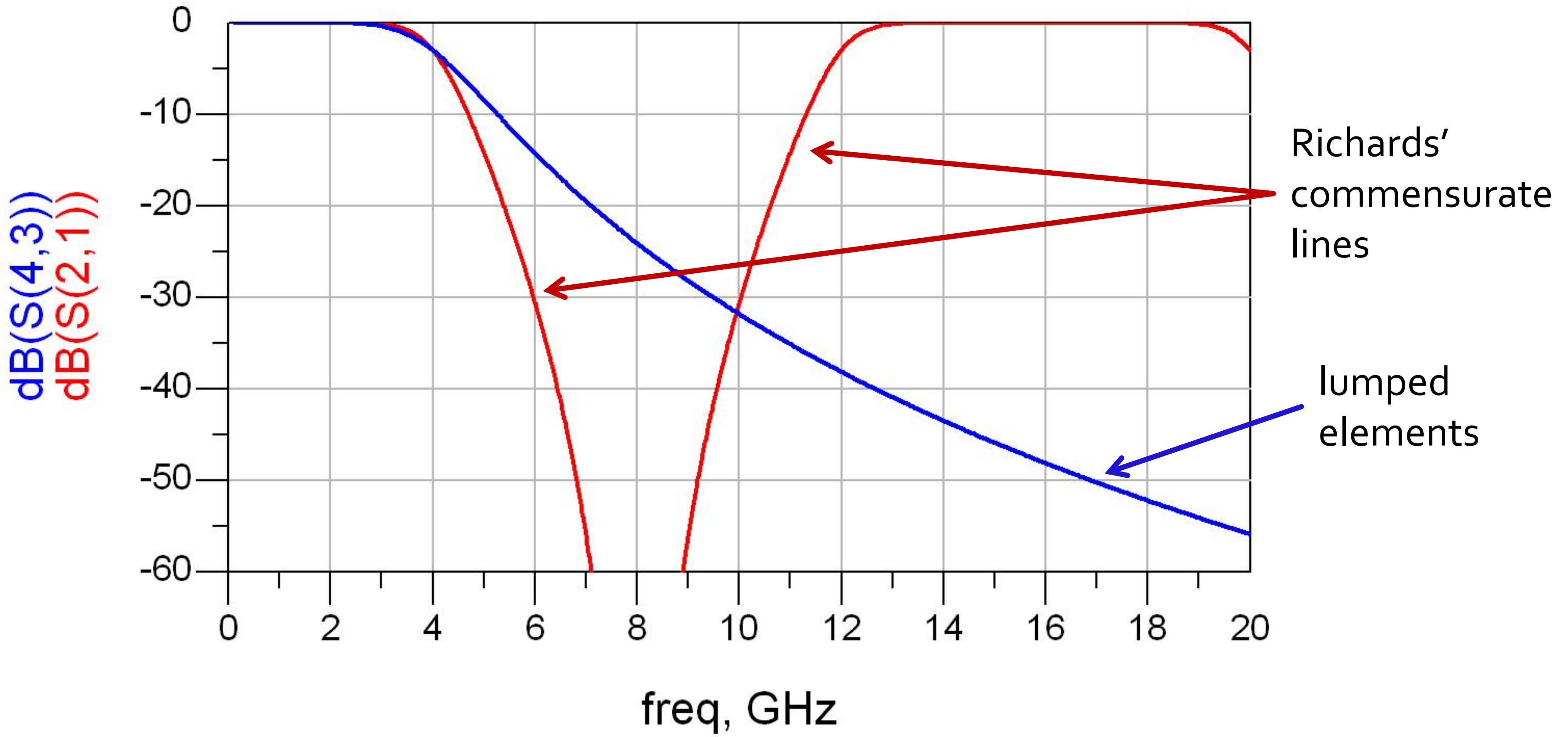
Richards' Transformation – ADS





Richards' Transformation

- Filters implemented with Richards' Transformation
  - beneficiate from the supplemental pole at $2 \cdot \omega_c$
  - have the major disadvantage of frequency periodicity, a supplemental non-periodic LPF must be inserted if needed



Equal-ripple prototype

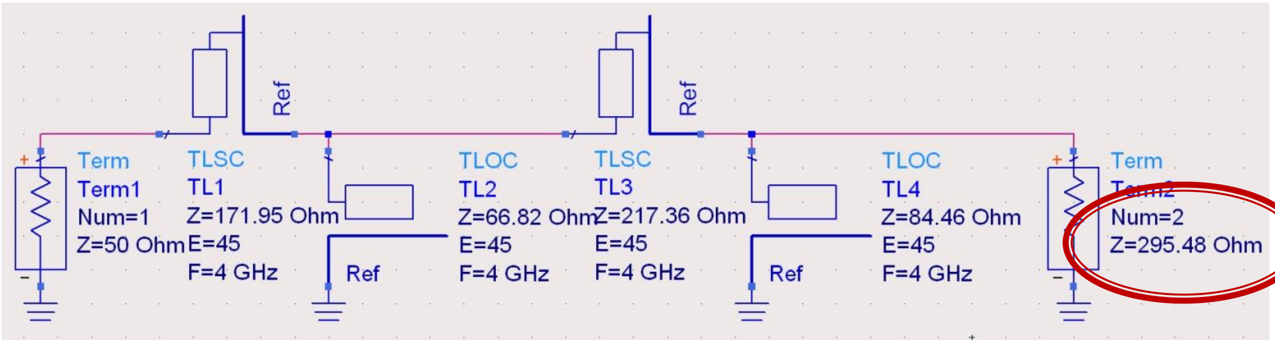
- For even N order of the filter (N = 2, 4, 6, 8 …) equal-ripple filters **must** closed by a non-standard load impedance $\mathbf{g_{N+1} \neq 1}$
- If the application doesn't allow this, supplemental impedance matching is required (quarter-wave transformer, binomial …) to $\mathbf{g_L = 1}$

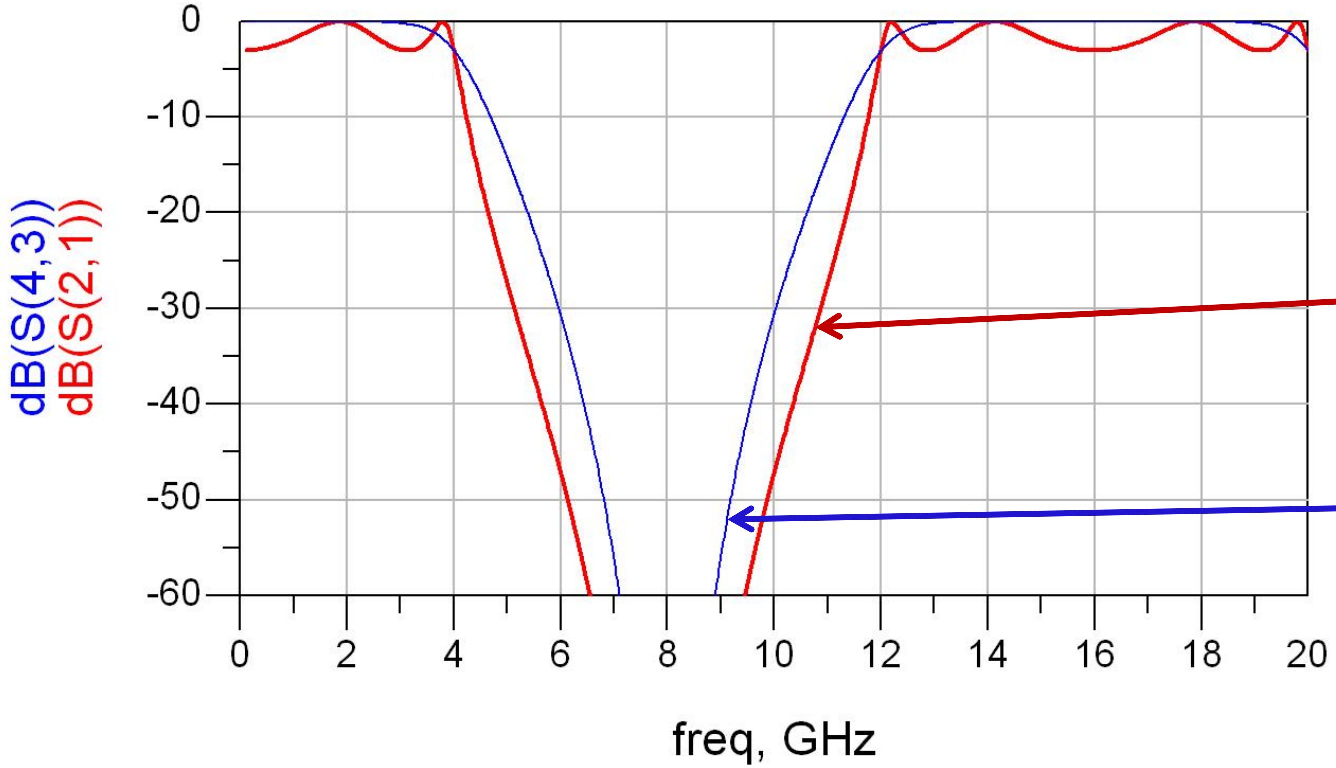
$$g_{N+1} \neq 1 \rightarrow R \neq R_0 \quad (50\Omega)$$

Observation: even order equal-ripple

- Same filter, 3dB equal-ripple
- 3dB equal-ripple tables or formulas:
  - $g_1 = 3.4389 = L_1$
  - $g_2 = 0.7483 = C_2$
  - $g_3 = 4.3471 = L_3$
  - $g_4 = 0.5920 = C_4$
  - $g_5 = 5.8095 = R_L$
- Line impedances
  - $Z_1 = 3.4389 \cdot 50\Omega = 171.945\Omega$ = series / short circuit
  - $Z_2 = 50\Omega / 0.7483 = 66.818\Omega$ = shunt / open circuit
  - $Z_3 = 4.3471 \cdot 50\Omega = 217.355\Omega$ = series / short circuit
  - $Z_4 = 50\Omega / 0.5920 = 84.459\Omega$ = shunt / open circuit
  - $RL = 5.8095 \cdot 50\Omega = 295.475\Omega$ = load

Even order equal-ripple – ADS



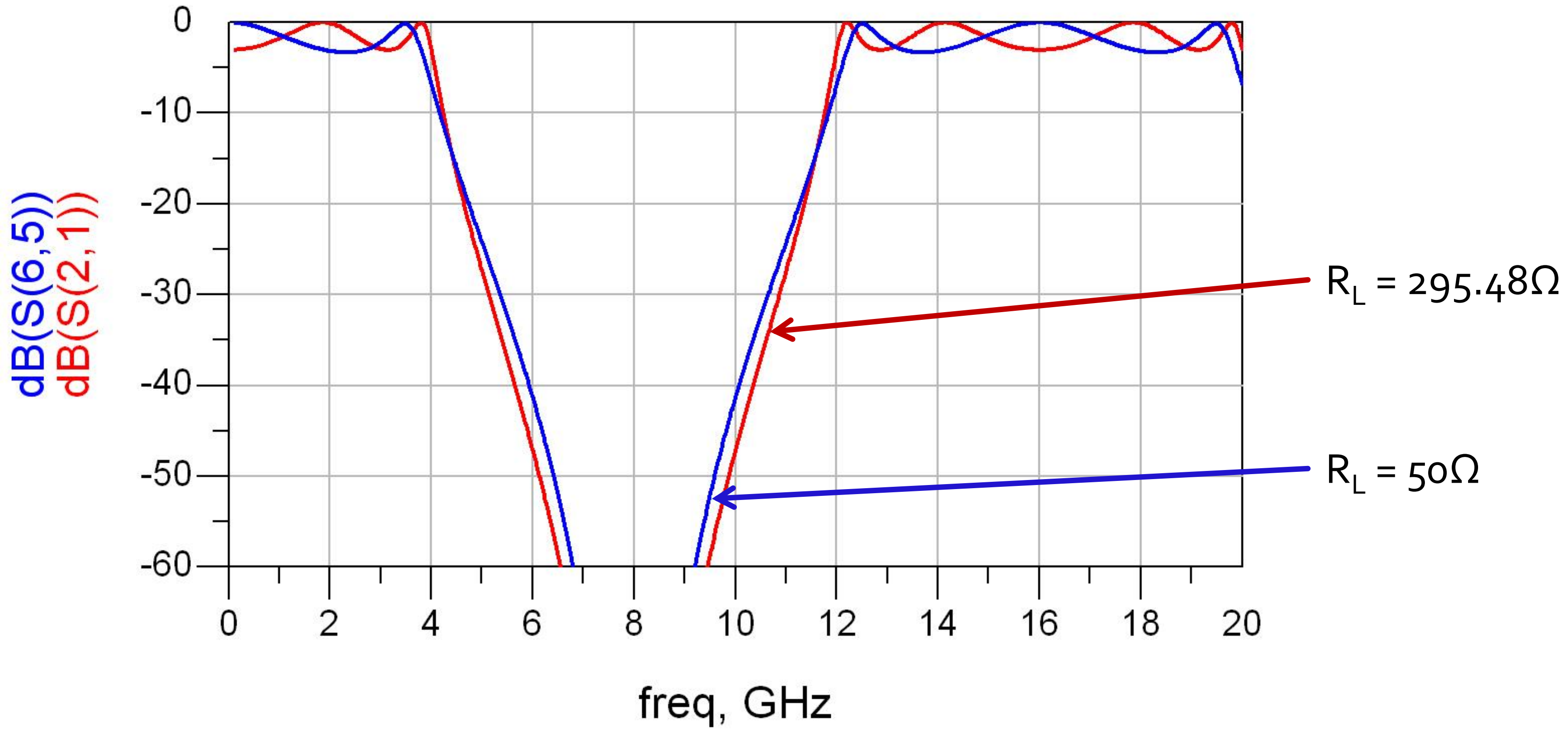


3dB equal-ripple (4th ord.)

maximum flat (4th ord)

Observation: even order equal-ripple

- Even order equal-ripple filters need output matching towards 50Ω for precise results. Example:



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